Abstract. We consider multiple tilings of $\mathbb{Z}$ by translates of a finite multiset $A$ of integers (called a tile). We say that a set of integers $T$ is an $A$-tiling of level $d$ if each integer can be written in exactly $d$ ways as the sum of an element of $T$ and an element of $A$. We find new exponential lower bounds on the longest period of $A$-tiling as a function of the diameter of $A$, which rejoin the exponential upper bounds given by Ruzsa (preprint, 2002) and Kolountzakis (2003). We also show the existence of tiles whose level semigroups have arbitrarily many generators (where the level semigroup of a tile $A$ is the set of integers $d$ such that $A$ admits a tiling of level $d$).