GLOBAL WORLD FUNCTIONS

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Abstract. Starting from the Amann-Conley-Zehnder finite reduction framework in the non-compact Viterbo’s version we discuss the existence of global generating function with a finite number of auxiliary parameters describing the two-points Characteristic Relation related to the geodesic problem in the Hamiltonian formalism. This applies both to Analytical Mechanics and to General Relativity – we construct a global object generalizing the World Function introduced by Synge, which is well-defined only locally. Whenever the auxiliary parameters can be fully removed, Synge’s World Function is restored.

1. Introduction

In the textbook by J. Synge [19] one can find the following definition:

Let \( P'(x') \) and \( P(x) \) be two points in the space-time, joined by a geodesic \( \Gamma \) with equations \( x^i = \xi^i(u) \) where \( u \) is a special\(^1\) parameter. Then the integral

\[
\Omega(P'P) = \Omega(x', x) = \frac{1}{2} \left( u_1 - u_0 \right) \int_{u_0}^{u_1} g_{ij} U^i U^j \, du
\]

taken along \( \Gamma \) with \( U^i = \frac{d\xi^i}{du} \), has a value independent of the particular special parameter chosen. If, as we shall suppose, the points \( P' \) and \( P \) determine a unique geodesic passing through them, then \( \Omega \) is a function of these two points. As a function of the eight variables \( x' \) and \( x \) we shall call it the world-function of space-time.

The World Function had a rather troubled history. The main criticism is that it has only a local meaning, and even in simple cases we cannot use it for global analysis. Really, Synge recognized this limitation a few lines after its definition.

\(^1\)special parameters are the representative elements of a class of parameters invariant by affine transformations.