On account of the complex structure of the middle ear and that it is more difficult to carry out experiments to test the nature of the mechanical components, the relevant data is hard to obtain, which has become an obstacle restricting analysis of the middle ear mechanic. Based on spatial structure and mechanical properties of ossicular chain, the paper has established functional relationship between load and members displacement with elastic principles and variation principles, in order that experimental results will be reflected in mechanical model. In the process of solving equations, we use the experimental data of a known special point or the various components function of statistical regression method and then combine them with the time shift function, so that the analytical solution of the various components will be achieved. The correctness of equation derived in this paper is verified by comparing the experimental data. So the model has provided a convenient way to obtain date in the future research analysis.

1. Introduction

Recently, with the rapid development of biological science, there is a growing concern about the research on human organs. So the research on middle ear structure and function has started in the ascendant. From different views, many scholars have been studying on middle-ear and sound conduction with different methods [1–20]. Abel et al. have obtained the geometry of ossicular in order to establish the finite element model in magnetic resonance microimaging [21]. Sun et al. have created middle ear FEM by a cross-calibration technique, and applied it to predicate stapes footplate displacement and the ossicular mechanics character of the human middle ear [22]. In the same year, in Takuji Koike’s study, a three-dimensional FEM of the human middle ear has been established, including features of the middle ear which were not considered in the previous model, that is, the ligaments,
tendons, I–S joint, loading of the cochlea, external auditory meatus, middle-ear cavities, and so forth. The validity of this model was confirmed by comparing the motion of the tympanic membrane and ossicular obtained by this model with the measurement data [23]. In Gan et al. study, they proposed a three-dimensional finite element model of the human ear. This model was constructed based on a complete set of histological section images of a left ear temporal bone. The FEM of the human ear was used to simulate ossicular joint to sound conduction affect [24].

Although these achievements are of great significant, the question of research process can not be ignored. For example, early research results were almost done on the cadaveric head or living animal; only observation experiment of some part in middle ear was done on normal human and lacked measurement of the whole member motion. Therefore, the quantity of measurement data was less than numerical simulation requirement. This limits the development numerical simulation method which was applied on middle ear research. In order to solve the problem, we have established mechanical model of ossicular and given its mathematical expression. The model can be used to test numerical simulation results.

2. Spatial Structure of Ossicular Chain

The ossicular chain is the smallest group of bones in human body, three bones together, they are the hammer (malleus), the anvil (incus), and the stirrup (stapes). These bones are connected and composition of a tiny link chain; see Figure 1. The ossicular chain comprises incudomalleolar joint and incudostapedial joint. Incudomalleolar joint is comprised of malleus and incus. Incudostapedial joint is comprised of incus and stapes. In general conditions, incudomalleolar joint and incudostapedial joint are immovable. Only in high sound pressure, the relative motion could occur. The ossicular chain is fixed the middle
Table 1: Parameters.

<table>
<thead>
<tr>
<th>Name</th>
<th>Length</th>
<th>Area</th>
<th>Displacement</th>
<th>Elastic modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anterior malleus ligament</td>
<td>$l_1$</td>
<td>$s_{l1}$</td>
<td>$u_1$</td>
<td>$k_{l1}$</td>
</tr>
<tr>
<td>Lateral malleus ligament</td>
<td>$l_2$</td>
<td>$s_{l2}$</td>
<td>$u_2$</td>
<td>$k_{l2}$</td>
</tr>
<tr>
<td>Superior malleus ligament</td>
<td>$l_3$</td>
<td>$s_{l3}$</td>
<td>$u_3$</td>
<td>$k_{l3}$</td>
</tr>
<tr>
<td>Tensor tympani muscle</td>
<td>$l_4$</td>
<td>$s_{l4}$</td>
<td>$u_4$</td>
<td>$k_{l4}$</td>
</tr>
<tr>
<td>Superior incus ligament</td>
<td>$l_5$</td>
<td>$s_{l5}$</td>
<td>$u_5$</td>
<td>$k_{l5}$</td>
</tr>
<tr>
<td>Posterior incus ligament</td>
<td>$l_6$</td>
<td>$s_{l6}$</td>
<td>$u_6$</td>
<td>$k_{l6}$</td>
</tr>
<tr>
<td>Stapedius muscle</td>
<td>$l_7$</td>
<td>$s_{l7}$</td>
<td>$u_7$</td>
<td>$k_{l7}$</td>
</tr>
</tbody>
</table>

Table 2: Parameters.

<table>
<thead>
<tr>
<th>Name</th>
<th>Area</th>
<th>Displacement</th>
<th>Elastic modulus</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stapes footplate</td>
<td>$s_{f8}$</td>
<td>$w_{10}$</td>
<td>$k_{st}$</td>
<td>$\rho_{ma}$</td>
</tr>
<tr>
<td>Malleus</td>
<td>$u_8$</td>
<td>$w_8$</td>
<td>—</td>
<td>$\rho_{ma}$</td>
</tr>
<tr>
<td>Incus</td>
<td>$u_9$</td>
<td>$w_9$</td>
<td>—</td>
<td>$\rho_{in}$</td>
</tr>
<tr>
<td>Stapes</td>
<td>$u_{10}$</td>
<td>$w_{10}$</td>
<td>—</td>
<td>$\rho_{st}$</td>
</tr>
<tr>
<td>Load on malleus</td>
<td>$w_{b</td>
<td>x=x}$</td>
<td>frequency</td>
<td>—</td>
</tr>
</tbody>
</table>

The length axis of ligament and muscle was defined to parallel coordinated axis in order to be simplified to derive formula. Tables 1 and 2 gave displacement and physics property of ligament, muscle, and ossicular chain. These ligaments and muscles can determine spatial position of ossicular chain in the middle ear cavity and affect motion states of ossicular chain under external dynamic loads.

3. Establishing Mechanics Model of Ossicular Chain

The length axis of ligament and muscle was defined to parallel coordinated axis in order to be simplified to derive formula. Tables 1 and 2 gave displacement and physics property of ligament, muscle, and ossicular chain.

Energy expressions of ossicular chain, ligament, and muscle be follows.

Malleus Kinetic Energy:

$$T_{ma} = \iiint_{\Omega} \frac{1}{2} \rho_{ma} \omega^2 w_8^2 d\Omega.$$  \hspace{1cm} (3.1)

Incus Kinetic Energy:

$$T_{in} = \iiint_{\Omega} \frac{1}{2} \rho_{in} \omega^2 w_9^2 d\Omega.$$  \hspace{1cm} (3.2)

Stapes Kinetic Energy:

$$T_{st} = \iiint_{\Omega} \frac{1}{2} \rho_{st} \omega^2 w_{10}^2 d\Omega.$$  \hspace{1cm} (3.3)
Elastic potential energy of anterior malleus ligament:

\[
E_{11} = \frac{1}{2} \iiint_{\Omega} k_{11} \left[ \left( \frac{\partial w_1}{\partial x} \right)^2 + \left( \frac{\partial w_1}{\partial y} \right)^2 + \left( \frac{\partial w_1}{\partial z} \right)^2 \right] \\
+ \frac{k_{11}}{2(1 + \mu)} \left[ \left( \frac{\partial w_1}{\partial x} + \frac{\partial w_1}{\partial y} \right)^2 + \left( \frac{\partial w_1}{\partial y} + \frac{\partial w_1}{\partial z} \right)^2 + \left( \frac{\partial w_1}{\partial x} + \frac{\partial w_1}{\partial z} \right)^2 \right] \, d\Omega.
\]  

(3.4)

Elastic potential energy of lateral malleus ligament:

\[
E_{12} = \frac{1}{2} \iiint_{\Omega} k_{12} \left[ \left( \frac{\partial w_2}{\partial x} \right)^2 + \left( \frac{\partial w_2}{\partial y} \right)^2 + \left( \frac{\partial w_2}{\partial z} \right)^2 \right] \\
+ \frac{k_{12}}{2(1 + \mu)} \left[ \left( \frac{\partial w_2}{\partial x} + \frac{\partial w_2}{\partial y} \right)^2 + \left( \frac{\partial w_2}{\partial y} + \frac{\partial w_2}{\partial z} \right)^2 + \left( \frac{\partial w_2}{\partial x} + \frac{\partial w_2}{\partial z} \right)^2 \right] \, d\Omega.
\]  

(3.5)

Elastic potential energy of superior malleus ligament:

\[
E_{13} = \frac{1}{2} \iiint_{\Omega} k_{13} \left[ \left( \frac{\partial w_3}{\partial x} \right)^2 + \left( \frac{\partial w_3}{\partial y} \right)^2 + \left( \frac{\partial w_3}{\partial z} \right)^2 \right] \\
+ \frac{k_{13}}{2(1 + \mu)} \left[ \left( \frac{\partial w_3}{\partial x} + \frac{\partial w_3}{\partial y} \right)^2 + \left( \frac{\partial w_3}{\partial y} + \frac{\partial w_3}{\partial z} \right)^2 + \left( \frac{\partial w_3}{\partial x} + \frac{\partial w_3}{\partial z} \right)^2 \right] \, d\Omega.
\]  

(3.6)

Elastic potential energy of tensor tympani muscle:

\[
E_{14} = \frac{1}{2} \iiint_{\Omega} k_{14} \left[ \left( \frac{\partial w_4}{\partial x} \right)^2 + \left( \frac{\partial w_4}{\partial y} \right)^2 + \left( \frac{\partial w_4}{\partial z} \right)^2 \right] \\
+ \frac{k_{14}}{2(1 + \mu)} \left[ \left( \frac{\partial w_4}{\partial x} + \frac{\partial w_4}{\partial y} \right)^2 + \left( \frac{\partial w_4}{\partial y} + \frac{\partial w_4}{\partial z} \right)^2 + \left( \frac{\partial w_4}{\partial x} + \frac{\partial w_4}{\partial z} \right)^2 \right] \, d\Omega.
\]  

(3.7)

Elastic potential energy of superior incus ligament:

\[
E_{15} = \frac{1}{2} \iiint_{\Omega} k_{15} \left[ \left( \frac{\partial w_5}{\partial x} \right)^2 + \left( \frac{\partial w_5}{\partial y} \right)^2 + \left( \frac{\partial w_5}{\partial z} \right)^2 \right] \\
+ \frac{k_{15}}{2(1 + \mu)} \left[ \left( \frac{\partial w_5}{\partial x} + \frac{\partial w_5}{\partial y} \right)^2 + \left( \frac{\partial w_5}{\partial y} + \frac{\partial w_5}{\partial z} \right)^2 + \left( \frac{\partial w_5}{\partial x} + \frac{\partial w_5}{\partial z} \right)^2 \right] \, d\Omega.
\]  

(3.8)
Elastic potential energy of posterior incus ligament:

\[ E_{16} = \frac{1}{2} \iiint_{\Omega} k_{16} \left[ \left( \frac{\partial w_6}{\partial x} \right)^2 + \left( \frac{\partial w_6}{\partial y} \right)^2 + \left( \frac{\partial w_6}{\partial z} \right)^2 \right] \]
\[ + \frac{k_{16}}{2(1+\mu)} \left[ \left( \frac{\partial w_6}{\partial x} + \frac{\partial w_5}{\partial y} \right)^2 + \left( \frac{\partial w_6}{\partial y} + \frac{\partial w_6}{\partial z} \right)^2 + \left( \frac{\partial w_6}{\partial x} + \frac{\partial w_6}{\partial z} \right)^2 \right] d\Omega. \] (3.9)

Elastic potential energy of stapedius muscle:

\[ E_{17} = \frac{1}{2} \iiint_{\Omega} k_{17} \left[ \left( \frac{\partial w_7}{\partial x} \right)^2 + \left( \frac{\partial w_7}{\partial y} \right)^2 + \left( \frac{\partial w_7}{\partial z} \right)^2 \right] \]
\[ + \frac{k_{17}}{2(1+\mu)} \left[ \left( \frac{\partial w_7}{\partial x} + \frac{\partial w_7}{\partial y} \right)^2 + \left( \frac{\partial w_7}{\partial y} + \frac{\partial w_7}{\partial z} \right)^2 + \left( \frac{\partial w_7}{\partial x} + \frac{\partial w_7}{\partial z} \right)^2 \right] d\Omega. \] (3.10)

Elastic potential energy of stapes footplate:

\[ E_{st} = \frac{1}{2} \iint_{s} k_{st} w_{10}^2 ds \] (3.11)

External loads do work:

\[ W = \frac{1}{2} P w_{8x} \bigg|_{x=q}. \] (3.12)

Structure strain energy:

\[ V = E_{11} + E_{12} + E_{13} + E_{14} + E_{15} + E_{16} + E_{17} + E_{st} - W. \] (3.13)

Potential energy of structure:

\[ U = V - T = E_{11} + E_{12} + E_{13} + E_{14} + E_{15} + E_{16} + E_{17} + E_{st} - W - T_{ma} - T_{in} - T_{st}. \] (3.14)

When the load was applied on ossicular chain, all members were together moving. Moreover displacement direction was the same in interface of two members. So displacement relationship in members is as follows.

Displacement Relationship between anterior malleus ligament and malleus:

\[ w_8 - w_1 | s_1 = 0. \] (3.15)

Displacement Relationship between lateral malleus ligament and malleus:

\[ w_8 - w_2 | s_2 = 0. \] (3.16)
Displacement Relationship between superior malleus ligament and malleus:

\[ w_8 - w_3 \mid s_3 = 0. \]  \hspace{2cm} (3.17)

Displacement Relationship between tensor tympani muscle and malleus:

\[ w_8 - w_4 \mid s_4 = 0. \]  \hspace{2cm} (3.18)

Displacement Relationship between superior incus ligament and incus:

\[ w_9 - w_5 \mid s_5 = 0. \]  \hspace{2cm} (3.19)

Displacement Relationship between posterior incus ligament and incus:

\[ w_9 - w_6 \mid s_6 = 0. \]  \hspace{2cm} (3.20)

Displacement Relationship between stapedius muscle and stapes:

\[ w_{10} - w_7 \mid s_7 = 0. \]  \hspace{2cm} (3.21)

Displacement Relationship between malleus and incus:

\[ w_9 - w_8 \mid s_8 = 0. \]  \hspace{2cm} (3.22)

Displacement Relationship between incus and stapes:

\[ w_{10} - w_9 \mid s_9 = 0, \]  \hspace{2cm} (3.23)

where \( s_1 \ldots s_9 \) is the area of interface.

**Boundary Constraint**

Displacement of the fixed end in anterior malleus ligament:

\[ w_1 \mid s_{10} = 0. \]  \hspace{2cm} (3.24)

Displacement of the fixed end in lateral malleus ligament:

\[ w_2 \mid s_{11} = 0. \]  \hspace{2cm} (3.25)

Displacement of the fixed end in superior malleus ligament:

\[ w_3 \mid s_{12} = 0. \]  \hspace{2cm} (3.26)
Displacement of the fixed end in tensor tympani muscle:

\[ w_4 \mid s_{13} = 0. \] (3.27)

Displacement of the fixed end in superior incus ligament:

\[ w_5 \mid s_{14} = 0. \] (3.28)

Displacement of the fixed end in posterior incus ligament:

\[ w_6 \mid s_{15} = 0. \] (3.29)

Displacement of the fixed end in stapedius muscle:

\[ w_7 \mid s_{16} = 0, \] (3.30)

where \( s_{10} \cdots s_{16} \) is the area of the fixed end in ligament and muscle.

Because elastic modulus of bone is ten thousand times than soft tissue, bone strain is less than soft tissue under loads and can be ignored. So bone displacement can be seen as rigid displacement caused by soft tissue motion. According to the published literature, when soft tissue strain is in elastic range, load and displacement will show linear relationship in terms of mechanical principle [26, 27]. Therefore, the whole structure displacement will show in linear relationship in the low stress conditions.

Based on the above analysis, the paper makes the two following assumptions:

(a) when load is fixed, the relationship between member displacement and spatial coordinate will be linear;

(b) when point is chosen, the point displacement will relate to load.

Based on the assumption, the composition of member displacement is coordinate and load. Moreover, the displacement of coordinate and load is independent. Member displacement expression:

Displacement of anterior malleus ligament

\[ w_1 = (a_{11}x + b_{11}y + c_{11}z + d_{11}) \cdot f. \] (3.31)

Displacement lateral malleus ligament:

\[ w_2 = (a_{21}x + b_{21}y + c_{21}z + d_{21}) \cdot f. \] (3.32)

Displacement of superior malleus ligament:

\[ w_3 = (a_{31}x + b_{31}y + c_{31}z + d_{31}) \cdot f. \] (3.33)
Displacement of tensor tympani muscle:

\[ w_4 = (a_{41} x + b_{41} y + c_{41} z + d_{41}) \cdot f. \]  \hspace{1cm} (3.34)

Displacement of superior incus ligament:

\[ w_5 = (a_{51} x + b_{51} y + c_{51} z + d_{51}) \cdot f. \]  \hspace{1cm} (3.35)

Displacement of posterior incus ligament:

\[ w_6 = (a_{61} x + b_{61} y + c_{61} z + d_{61}) \cdot f. \]  \hspace{1cm} (3.36)

Displacement of stapedius muscle:

\[ w_7 = (a_{71} x + b_{71} y + c_{71} z + d_{71}) \cdot f. \]  \hspace{1cm} (3.37)

Malleus displacement:

\[ w_8 = \begin{cases} 
(a_{81} x + b_{81} y + c_{81} z + d_{81}) \cdot f & x < x_1, \\
(a_{82} x + b_{82} y + c_{82} z + d_{82}) \cdot f & x > x_1.
\end{cases} \]  \hspace{1cm} (3.38)

Incus displacement:

\[ w_8 = \begin{cases} 
(a_{91} x + b_{91} y + c_{91} z + d_{91}) \cdot f & x < x_2, \\
(a_{92} x + b_{92} y + c_{92} z + d_{92}) \cdot f & x > x_2.
\end{cases} \]  \hspace{1cm} (3.39)

Stapes displacement:

\[ w_{10} = (a_{101} x + b_{101} y + c_{101} z + d_{101}) \cdot f, \]  \hspace{1cm} (3.40)

where \( x_1, x_2 \) is the interface of variable malleus section or variable incus section. \( a, b, c, d \) are coefficient and they have subscript. \( f \) is a function to relate with load.

\( w_1, w_2 \cdots w_7 \), expression including \( f \) is solved by knowing boundary conditions and testing results of key points. And then based on relationship of ossicular and ligament, \( w_8, w_9, w_{10} \) expression including \( f \) is solved. Finally all expressions are substituted to potential energy of structure \( U \). Applying on variation principles to \( f \), getting \( \delta U = 0, f \) is solved and is substituted to displacement expression of members (ossicular, ligament and muscle).

Another method is that displacement function expressions of member were obtained by linear regression based on similar results. The geometrical size of finite element model based on the images of CT in healthy human by Zhongshan Hospital affiliated to Fudan University. All patients were scanned with a 64-slice multiple spiral CT scanner (GE lightspeed VCT) using the following parameters: 0.625 mm collimation, 0.42 second per
Table 3: Material properties of ossicular chain [22].

<table>
<thead>
<tr>
<th>Name</th>
<th>Density (kg/m³)</th>
<th>Elastic modulus (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>malleus</td>
<td></td>
<td></td>
</tr>
<tr>
<td>for head</td>
<td>2.55e3</td>
<td>1.41e10</td>
</tr>
<tr>
<td>for neck</td>
<td>4.53e3</td>
<td>1.41e10</td>
</tr>
<tr>
<td>for handle</td>
<td>3.7e3</td>
<td>1.41e10</td>
</tr>
<tr>
<td>for body</td>
<td>2.36e3</td>
<td>1.41e10</td>
</tr>
<tr>
<td>Superior ligament</td>
<td>2.5e3</td>
<td>4.9e4</td>
</tr>
<tr>
<td>lateral ligament</td>
<td>2.5e3</td>
<td>6.7e4</td>
</tr>
<tr>
<td>anterior ligament</td>
<td>2.5e3</td>
<td>2.1e6</td>
</tr>
<tr>
<td>incus</td>
<td></td>
<td></td>
</tr>
<tr>
<td>for short process</td>
<td>2.26e3</td>
<td>1.41e10</td>
</tr>
<tr>
<td>for long process</td>
<td>5.08e3</td>
<td>1.41e10</td>
</tr>
<tr>
<td>superior ligament</td>
<td>2.5e3</td>
<td>4.9e4</td>
</tr>
<tr>
<td>posterior ligament</td>
<td>2.5e3</td>
<td>6.5e5</td>
</tr>
<tr>
<td>Stapes</td>
<td>2.2e3</td>
<td>1.41e10</td>
</tr>
<tr>
<td>tympani muscle</td>
<td>2.5e3</td>
<td>2.6e6</td>
</tr>
<tr>
<td>stapedius muscle</td>
<td>2.5e3</td>
<td>5.2e5</td>
</tr>
<tr>
<td>cochlear impedance</td>
<td>—</td>
<td>60</td>
</tr>
</tbody>
</table>

wrap, 0.625 mm reconstruction slice thickness, and 0.5–0.625 mm reconstruction increment. The scan images were managed with self-edit program to form a geometric model. The model was reconstructed for optimizing meshes, setting boundary conditions and material properties in ANSYS. Finally, the 3D fluid-solid coupling finite element model of ossicular chain was established successfully; see Figure 2. Finite element model of ossicular chain referred to Zhai et al.’s results [28]. Ligament dimension in ossicular chain referred to Lemmerling et al.’s results [29]. The dimension of stapedius muscle and tensor tympani muscle referred to Beer et al.’s data [30]. Table 3 gave material properties of ossicular chain, which referred to Sun et al.’s data [22].

Figure 3 gave the simulation result of the displacement in different sound pressure levels: 50 dB, 70 dB, 90 dB, 105 dB, and 120 dB. The conclusions were obtained from similar results. Members were together moving, and moving direction was shown in Figure 3. Displacement direction was the same in interface of members in Figure 3; The result tested the hypothesis of theoretical derivation to be valid. Coordinate function of member displacement of linear regression by similar data was done.

Then the function was multiplied by $f$ function. Expression is as follows.

Displacement of anterior malleus ligament:

$$w_1 = (0.001351x - 0.000685y + 0.002547z - 0.00000044) \cdot f.$$  \hspace{1cm} (3.41)

Displacement of lateral malleus ligament:

$$w_2 = (-0.002916x + 0.001829y + 0.000761z + 0.0000134) \cdot f.$$  \hspace{1cm} (3.42)
Displacement of superior malleus ligament:
\[ w_3 = (-0.008318x - 0.007172y - 0.0025405z + 0.0000997) \cdot f. \] (3.43)

Displacement of tensor tympani muscle:
\[ w_4 = (-0.000432x - 0.001813y + 0.000575z + 0.00000506) \cdot f. \] (3.44)

Displacement of superior incus ligament:
\[ w_5 = (-0.006154x - 0.005452y + 0.000487z + 0.0000736) \cdot f. \] (3.45)

Displacement of posterior incus ligament:
\[ w_6 = (0.000493x + 0.001017y - 0.003073z + 0.0000159) \cdot f. \] (3.46)

Displacement of stapedius muscle:
\[ w_7 = (-0.000633x - 0.00299y - 0.007289z + 0.0000131) \cdot f. \] (3.47)
Malleus displacement:

\[
\begin{cases} 
-w_8 = \left\{ \begin{array}{ll} 
(0.001699x - 0.000251y + 0.000848z + 0.0000572) \cdot f & x < 0.00267 \\
(0.001409x + 0.000777y - 0.000617z - 0.0000392) \cdot f & x > 0.00267,
\end{array} \right. 
\end{cases}
\] (3.48)

Incus displacement:

\[
\begin{cases} 
-w_9 = \left\{ \begin{array}{ll} 
(-0.001567x + 0.001242y + 0.000268z + 0.0000207) \cdot f & x < 0.00256 \\
(0.001020x + 0.001420y - 0.000488z - 0.000004510) \cdot f & x > 0.00256,
\end{array} \right. 
\end{cases}
\] (3.49)

Stapes displacement:

\[
w_{10} = (-0.000204x + 0.000541y - 0.002076z + 0.00000566) \cdot f. 
\] (3.50)

Front displacements were substituted to energy equation. Finally energy equations were substituted to potential energy of structure \(U\). Applying on variation principles to \(f\), getting \(\delta U = 0\), \(f\) was solved and was substituted to displacement expression of members (ossicular, ligament, and muscle).

### 4. Example

The curve that displacement of umbo and footplate centre was changed with frequency under 105 dB was computed. Load and displacement equations of linear regression by similar data were substituted to potential energy of structure \(U\). Applying on variation principles to \(f\), getting \(\delta U = 0\), \(f\) was solved:

\[
f = \frac{1.56 \times 10^{-10}}{2 \times 3.39 \times 10^{-9} - 2 \times 1.01 \times 10^{-15} \omega^2}. 
\] (4.1)

\(f\) and relating to coordinate were substituted to displacement expression of umbo and footplate centre Expression is as follows.

Footplate centre displacement:

\[
w_{10} = \frac{2.63 \times 10^{-16}}{2 \times 3.39 \times 10^{-9} - 2 \times 1.01 \times 10^{-15} \omega^2}. 
\] (4.2)

Umbo displacement:

\[
w_9 = \frac{3.83 \times 10^{-16}}{2 \times 3.39 \times 10^{-9} - 2 \times 1.01 \times 10^{-15} \omega^2}. 
\] (4.3)

Computing results were compared with the data in published paper; see Figure 4 [31]. The curves of analytical solution were similar to those of experiment. This showed that analytical solution is valid. The difference of two curves was little in low and high frequency and big in 700–2000 Hz from Figure 4. It was caused by geometry size of model.
5. Conclusion

The equation in the paper was one of the valid methods of obtaining experiment data. It could determinate function relationship based on experiment data of special point, and then experiment data of unknown point were computed according to the function relationship. This could get a large number of experiment data and solve difficulty of getting data. In addition, the model can analyze the effect which was caused by members’ injury in motion or material changes to stapes displacement. For example, these problems were some lesions of middle ear, joint injury, ligament sclerosis, or tendon sclerosis.

Acknowledgment

The authors gratefully acknowledge the Major Project of Shanghai Scientific Committee for Fundamental Research (no. 08jc1404700).

References


