Research Article

Modeling and Fuzzy PDC Control and Its Application to an Oscillatory TLP Structure

Cheng-Wu Chen

Department of Logistics Management, Shu-Te University, 59 Hun Shan Road, Yen Chau, Kaohsiung County, 82445, Taiwan

Correspondence should be addressed to Cheng-Wu Chen, cwchen@stu.edu.tw

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An analytical solution is derived to describe the wave-induced flow field and surge motion of a deformable platform structure controlled with fuzzy controllers in an oceanic environment. In the controller design procedure, a parallel distributed compensation (PDC) scheme is utilized to construct a global fuzzy logic controller by blending all local state feedback controllers. The Lyapunov method is used to carry out stability analysis of a real system structure. The corresponding boundary value problems are then incorporated into scattering and radiation problems. These are analytically solved, based on the separation of variables, to obtain a series of solutions showing the harmonic incident wave motion and surge motion. The dependence of the wave-induced flow field and its resonant frequency on wave characteristics and structural properties including platform width, thickness and mass can thus be drawn with a parametric approach. The wave-induced displacement of the surge motion is determined from these mathematical models. The vibration of the floating structure and mechanical motion caused by the wave force are also discussed analytically based on fuzzy logic theory and the mathematical framework to find the decay in amplitude of the surge motion in the tension leg platform (TLP) system. The expected effects of the damping in amplitude of the surge motion due to the control force on the structural response are obvious.

1. Introduction

The supply and demand for the products of natural energy have been rapidly increasing in recent years. This has motivated oil/gas producers to explore even the ocean depths to obtain these resources. This has necessitated the in-depth study and analysis of deep water structures, such as the tension leg platform (TLP), which are particularly suited for water depths greater than 300 meters [1–4]. A TLP is a vertically moored floating structure often used for oil exploration and ocean thermal energy conversion (OTEC) [5]. This type of structure is particularly suited for water depths greater than 300 meters. They have also been used in oceanic environments as floating breakwaters [6]. The platform is permanently moored by means of “tethers” or “tendons” grouped at each of the structure’s corners.
Each group of tethers forms a “tension leg.” A typical tension leg platform usually includes a floating body which is anchored to the seafloor by the pretensioned legs. This semi-submerged floating body may be comprised of different materials with a working platform on top. Previous studies have introduced simplified models considering the surge motion of an impermeable structure with linearly elastic, pretensioned legs \cite{7–10}. In studies of actual engineering applications the motion of the tethers is simplified as on-line rigid-body motion proportional to the top platform for estimating the pretension force without the drag effect. The resultant drag effect on the TLP system gives rise to an important issue related to solution types for the TLP system. Lee and Wang present a complete analytical solution for the dynamic behavior of both the platform and tethers in the TLP system \cite{11}. In their study the TLP is subjected to wave-induced surge motion and the flow-induced drag motion, where the drag surge motion of the TLP is considered. Some research results show that the tether drag significantly reduces the response of the TLPs \cite{11, 12}. In those studies, however, only surge motion was considered for the interaction between the wave and the platform structure.

The reduction of the vibration of structures by passive methods such as mass, stiffness, and damping has long been a subject of study in the fields of oceanic, mechanical, and civil engineering. Interest in the usage of tuned mass dampers and tuned liquid column dampers for reducing vibration in oceanic structures has also increased (see \cite{12, 13} and the references therein). There have been numerous applications of passive damping in structural systems leading to a significant improvement in stability. Passive damping is regarded as a simple, low-cost, and easily implemented method for reducing structural vibrations. Another advantage of passive damping is that it ensures that the control system can remain stable when uncertainties or disturbances exist in the structural system. However, TLP systems have infinite numbers of resonant frequencies (or structural modes); so to meet all exigencies one would need an equal number of absorbers to minimize each mode. This is clearly impractical. Therefore, this study develops an innovative approach for achieving an analytical methodology of fuzzy control for the stabilization of TLP systems. In the remainder of this article, we examine a methodology for designing fuzzy controllers which uses the Lyapunov criterion to ensure the stability of the controlled systems.

Much work has been carried out in both industry and academic fields since Zadeh \cite{14} first proposed using a linguistic approach to simulate the thought processes and judgment of human beings in fuzzy logic (see \cite{15–17} and the references therein). However, there are few available mathematical theories or systematic designs for any of these approaches. Takagi and Sugeno \cite{18} proposed a new fuzzy inference system, called the Takagi-Sugeno (T-S) fuzzy model, which combines the flexibility of fuzzy logic theory and rigorous mathematical analysis tools into a unified framework. Linear models are employed for the consequent parts which makes it simple to use conventional linear system theory for the analysis. The advantage of T-S fuzzy modeling is its greater accuracy, dimensionality, and also the simplification of the structure of nonlinear systems. It provides an effective representation of complex nonlinear systems (see \cite{19–24} and the references therein).

In recent years, many control methods have been proposed and there are increasing research activities in the field of structural systems. These methods include optimal control, fuzzy control, pole placement, and sliding mode control (see \cite{25–33} and the references therein). Hsiao et al. proposed a model-based fuzzy control method for dealing with the structure responses and stabilization analysis of passive tuned mass damper systems \cite{33}. The control of TLPs, however, is more complicated, involving boundary value problems and structure resonant response. The analysis of the stability and stabilization problem of TLPs
has not often been discussed. In this study, we apply a fuzzy control technique as well as a T-S fuzzy model to deal with TLP stability problems. A suitable mathematical model of the TLP system and the interaction between a deformable floating structure and surface wave motion are discussed incorporating a partial differential equation as well as fuzzy logic theory. The effects on the dragged surge vibration of a TLP system and the interaction between wave and structure of a set of PDC fuzzy controllers on a platform are considered. It is the purpose of this study to carry out stability analysis and examine the influence of the fuzzy controller on platform motion when the true behavior of the deformable TLP is included in the solution. Along with the solution which is presented as a comparison of the responses between the platform with and without fuzzy controllers, the influences of parameters on the floating structure, the water wave, and the fuzzy controller system are also studied.

2. Boundary Value Problems

The flow field is first divided into three columns with two artificial boundaries at $x = -b$ and $x = b$, as seen in Figure 1, in which $b$ denotes platform width (thus the platform width is $2b$) and $d$ is the platform draft. In Region I, $-\infty < x < -b$, the total velocity potential $\phi_I$ consists of incident waves $\phi_i$, scattered waves $\phi_{IS}$, motion radiation waves $\phi_{IW}$, and vibration radiation waves $\phi_{IV}$. In Region II, $-b < x < b$, and in Region III, $b < x < \infty$, the total velocity potentials $\phi_{II}$ and $\phi_{III}$ consist of both scattered, $\phi_{IS}$ and $\phi_{IIS}$, and radiated waves, $\phi_{IIW}$, $\phi_{IWW}$, $\phi_{III}$, and $\phi_{IIIW}$, as seen in Figure 1, where the subscript “s” denotes the scattering problem, “w” the wave-maker (i.e., primitive radiation) problem induced by surge motion, and “v” the vibration radiation problem induced by the deformable platform. The corresponding boundary value problems for the scattering and radiation problems are given in Figure 2, where the displacement of the surge motion with unknown amplitude $S$ is given by $X = Se^{-\sigma t}$, and the deformation of the platform on $z$-axis is defined as $\varsigma$, which is function of $x$ and $t$.

For a discussion of the kinematic boundary condition, dynamic boundary condition and radiation condition please refer to the stability analysis in [1].
The identification approach is appropriate for plants that are unable to be represented by analytical models. In some cases, because nonlinear dynamic models including algebraic equations for structure systems can be readily obtained, the second approach to derive a fuzzy model is more suitable. This study focuses on using the following Takagi's, Sugeno's, and Kang's outstanding work which individual rules are combined to describe the global behavior of the system.

Recently, fuzzy-rule-based modeling has become an active research field because of its unique merits for solving complex nonlinear system identification and control problems. Unlike traditional modeling, fuzzy rule-based modeling is essentially a multimodel approach in which individual rules are combined to describe the global behavior of the system [19, 34]. Therefore, fuzzy modeling is employed to represent a structural system in order to simplify the controller design problem.

To ensure the stability of the structural system, Takagi-Sugeno (T-S) fuzzy models and the stability analysis are recalled. There are two approaches for constructing fuzzy models mentioned in literature [35]:

1. identification (fuzzy modeling) using input-output data,
2. derivation from given nonlinear interconnected system equations.

There has been extensive work done on fuzzy modeling using input-output data following Takagi’s, Sugeno’s, and Kang’s outstanding work (see [35], and the references therein). The procedure is mainly composed of two parts: structure identification and parameter identification. The identification approach is appropriate for plants that are unable to be represented by analytical models. In some cases, because nonlinear dynamic models including algebraic equations for structure systems can be readily obtained, the second approach to derive a fuzzy model is more suitable. This study focuses on using the
second approach to constructing fuzzy models. The linear feedback control techniques can be utilized for feedback stabilization in the T-S type fuzzy model. The procedure is as follows: first, the plant is represented by a Takagi-Sugeno-type fuzzy model. In this type of fuzzy model, local dynamics in different state-space regions are represented by linear models. The overall model of the system is achieved by fuzzy “blending” of these linear models through nonlinear fuzzy membership functions.

3.2. Stability of Fuzzy Control

The momentum equation can be obtained from the motion of the floating structure, extensively derived from Newton’s second law. Assume that the momentum equation of a TLP system controlled by actuators can be characterized by the following differential equation:

\[ M\ddot{X}(t) = BU(t) - M\phi(t), \quad (3.1) \]

where \( \ddot{X}(t) = [\ddot{x}_1(t), \ddot{x}_2(t) \cdots \ddot{x}_n(t)] \in \mathbb{R}^n \) is an n-vector; \( \dot{X}(t) \), \( \ddot{X}(t) \), and \( X(t) \) are acceleration, velocity and displacement vectors, respectively; \( B \) is an \( (n \times m) \) matrix denoting the locations of m control forces; \( U(t) \) corresponds to the actuator forces (generated via the active tendon system or an active mass damper). This is only a static model. \( M\phi(t) \) is a wave-induced external force which can be expressed by

\[ M\phi(t) = F_{wx} - F_{Tx}, \quad (3.2) \]

where \( F_{wx} \) is the horizontal wave force acting on the both sides of the structure, and \( F_{Tx} \) is the horizontal component of the static (or the pretensioned) tension applied by the tension legs. The static tension is given by \( F_{Tx} = f_\xi \), in which \( f \) is the stiffness coefficient defined by \( f = 2pgb(d_2 - d_0)/(h - d_2) \). The structure parameters \( d_0 \) and \( d_2 \) are the drafts of the structure before and after applying pretension, respectively. According to the formula, \( d_0 \) increases as the structure’s mass increases, and the pretension stiffness \( f \) is inversely proportional to the mass for the specific structural geometry.

For the controller design, the standard first-order state equation corresponding to (3.2) is obtained by [36]

\[ \dot{X}(t) = AX(t) + BU(t) + E\phi(t), \quad (3.3) \]

where

\[ X(t) = \begin{bmatrix} \ddot{x}(t) \\ \dot{x}(t) \end{bmatrix}, \quad A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M^{-1}B \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ -P \end{bmatrix}. \quad (3.4) \]

To ensure the stability of the TLP system, T-S fuzzy models and some stability analysis are utilized. To design fuzzy controllers, the structural systems are represented by Takagi-Sugeno fuzzy models. The concept of PDC is employed to determine the structures of the fuzzy controllers from the T-S fuzzy models in this section. Some detailed steps for designing
PDC fuzzy controllers are described in literature [35, 37]. First, the \( i \)th rule for the T-S fuzzy model, representing the structural system (3.3), is as follows.

Rule \( i \): IF \( x_1(t) \) is \( M_{i1} \) and \( \cdots \) and \( x_p(t) \) is \( M_{ip} \),

\[
\text{THEN } \dot{X}(t) = A_iX(t) + B_iU(t) + E_i\phi(t),
\]

(3.5)

where \( i = 1, 2, \ldots, r \) and \( r \) is the rule number; \( X(t) \) is the state vector; \( M_{ip} \) (\( p = 1, 2, \ldots, g \)) are the fuzzy sets; \( x_1(t), x_2(t), \ldots, x_p(t) \) are the premise variables. By using the fuzzy inference method with a singleton fuzzifier, product inference, and center average defuzzifier, the dynamic fuzzy model (3.5) can be expressed as follows:

\[
\dot{X}(t) = \frac{\sum_{i=1}^{r} w_i(t) [A_iX(t) + B_iU(t) + E_i\phi(t)]}{\sum_{i=1}^{r} w_i(t)} = \sum_{i=1}^{r} h_i(t)(A_iX(t) + B_iU(t)) + E_i\phi(t)
\]

(3.6)

where

\[
w_i(t) = \prod_{p=1}^{g} M_{ip}(x_p(t)), \quad h_i(t) = \frac{w_i(t)}{\sum_{i=1}^{r} w_i(t)}.
\]

(3.7)

\( M_{ip}(x_p(t)) \) is the grade of membership of \( x_p(t) \) in \( M_{ip} \). It is assumed that

\[
w_i(t) \geq 0, \quad i = 1, 2, \ldots, r; \quad \sum_{i=1}^{r} w_i(t) > 0
\]

(3.8)

for all \( t \). Therefore, \( h_i(t) \geq 0 \) and \( \sum_{i=1}^{r} h_i(t) = 1 \) for all \( t \).

The PDC is adopted to design a global controller for the T-S fuzzy model (3.5). Using the same premise as (3.5), the \( i \)th rule of the FLC can be obtained as follows.

Controller Rule \( i \): IF \( x_1(t) \) is \( M_{i1} \) and \( \cdots \) and \( x_g(t) \) is \( M_{ig} \),

\[
\text{THEN } U(t) = -K_iX(t) \quad (i = 1, 2, \ldots, r),
\]

(3.9)

where \( K_i \) is the local feedback gain vector in the \( i \)th subspace. The final model-based fuzzy controller is analytically represented by

\[
U(t) = -\frac{\sum_{i=1}^{r} w_i(t)K_iX(t)}{\sum_{i=1}^{r} w_i(t)} = -\sum_{i=1}^{r} h_i(t)K_iX(t).
\]

(3.10)

The overall closed-loop controlled system obtained by combining (3.6) and (3.10) is

\[
\dot{X}(t) = \sum_{i=1}^{r} \sum_{t=1}^{r} h_i(t)h_i(t)[(A_i - B_iK_i)X(t)] + E_i\phi(t).
\]

(3.11)

A typical stability condition for a fuzzy system (3.11) is proposed here.
Table 1: Input data for case study of surge motion calculation.

<table>
<thead>
<tr>
<th>Initial wave conditions</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave frequency $\sigma$</td>
<td></td>
</tr>
<tr>
<td>Wave amplitude $A_i$</td>
<td>0.5 m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Environmental conditions</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Water depth $h$</td>
<td>30 m</td>
</tr>
<tr>
<td>Gravitational force $g$</td>
<td>9.8 m/s²</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Structural parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass $M$</td>
<td>10, 100, and 1000 kg</td>
</tr>
<tr>
<td>Draft $d$</td>
<td>2 m</td>
</tr>
<tr>
<td>Width $b$</td>
<td>10 m</td>
</tr>
<tr>
<td>Young’s modulus $E$</td>
<td>$1 \times 10^7$ N/m²</td>
</tr>
<tr>
<td>Mass moment of inertia</td>
<td>$10^{-2}$ m⁴</td>
</tr>
</tbody>
</table>

**Theorem 3.1.** The equilibrium point of the fuzzy control system (3.11) is stable in the large if there exist a common positive definite matrix $P$ and feedback gains $K$ such that the following two inequalities are satisfied:

\[
\begin{align*}
(A_i - B_i K_i)^T P + P (A_i - B_i K_i) + \frac{1}{\eta^2} P E_i E_i^T P < 0, \\
\left(\frac{(A_i - B_i K_i) + (A_i - B_i K_i)}{2}\right)^T P + P \left(\frac{(A_i - B_i K_i) + (A_i - B_i K_i)}{2}\right) + \frac{1}{\eta^2} P E_i E_i^T P < 0
\end{align*}
\]

(3.12)

with $P = P^T > 0$, for $i \leq l \leq r$ and $i = 1, 2, \ldots, r$.

The proof is lengthy and can be derived by the similar approach by Hsiao et al. [33, 37–39] and Yeh et al. [31]. Therefore, the proof is removed in this paper.

**4. Numerical Results**

In the physical world, engineers measure and observe the effects induced by embedded structures. They might conduct an experiment or carry out real scale numerical simulations. We do not especially consider engineering oriented issues. Instead, we utilize sensitive concepts to explain the effects of physical parameters on a natural transformation. The dynamic response of a platform in a TLP system is dependent on a large set of parameters that include the characteristics of the waves and the material properties of the structure. Several parameters are particularly important for systematic consideration in TLP stability and stabilization problems. These include the amplitude and frequency of the waves (wave characteristics) as well as the structural properties of the platform (including draft, width, thickness mass, etc.). Only platform mass is considered in the following discussion. The parameters are given in Table 1. A plot of the reflection coefficient versus dimensionless wave frequency is shown in Figure 3. It can be seen from this plot that the reflection coefficient increases with the platform mass. On the other hand, as can be seen in Figure 4, the transmission coefficient decreases as the platform mass increases. Obviously, this is in good agreement with the opposing relationship between these two coefficients, which can be defined as the ratio of the amplitude
of the reflected/transmitted wave and the amplitude of the incident wave. Figure 5 shows profiles of surge motion for structures of varied mass but fixed 0.5 m amplitude wave fluctuation. Generally speaking, structural displacement increases with decreasing mass. It is found that the greater the dimensionless wave frequency is, the greater surge motion will be. The calculated maximum displacement of surge motion for a 1000 kg platform mass is estimated to be approximately 60 times the wave amplitude. The dimensionless horizontal platform displacement becomes the largest when in the range of $\sigma^2 h/g = 0.4$ to 1.0, with values of 58, 52, and 47. The resonance frequency also shifts to a lower value when the mass of the structure increases, which corresponds to a decreasing stiffness coefficient. It is commonly known that an increase in the structural mass reduces the equivalent stiffness caused by increasing draft. Therefore, platform response is enhanced in cases where the mass of the structure increases. This occurs because of the weaker pretension stiffness. In the TLP system, frequencies become smaller as the mass of the system increases. In addition, the control force acts to significantly mitigate the amplitude of the surge motion when the wave-structure interaction is taken
into account. Table 2 shows the periodic wave-induced structure responses with respect to the several wave conditions shown in Figures 6 and 7. It can be seen by looking at Figure 6 that the state of displacement and velocity of TLP systems will be stabilized under external waves. It can be seen in Figure 7 that the control forces employed by PDC fuzzy schemes can stabilize systems with harmonic inputs. A look at these graphs shows that the TLP system is stable. Note that the trajectories for the three different conditions starting from the initial nonzero states all approach close to the origin under arbitrary harmonic excitation. This demonstrates that the effects of the control force have acted on the TLP system to stabilize the oceanic structure. A comparison of the results for these three figures shows the periodic wave fluctuation. The implication is that the control force enhances the decay of the structural displacement and motion velocity over time. The results of analysis show that despite the fact that much progress has been made in successfully applying a passive control force to TLP system, many basic issues remain that need to be further addressed.
Table 2: Input data for three parametric cases.

<table>
<thead>
<tr>
<th>Subsystem 1</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Wave period $T$</td>
<td>10 s</td>
</tr>
<tr>
<td>Structure mass $M$</td>
<td>100 kg</td>
</tr>
<tr>
<td>Structure width</td>
<td>4 m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subsystem 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave period $T$</td>
<td>12 s</td>
</tr>
<tr>
<td>Structure mass $M$</td>
<td>200 kg</td>
</tr>
<tr>
<td>Structure width</td>
<td>6 m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subsystem 3</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave period $T$</td>
<td>8 s</td>
</tr>
<tr>
<td>Structure mass $M$</td>
<td>150 kg</td>
</tr>
<tr>
<td>Structure width</td>
<td>2 m</td>
</tr>
</tbody>
</table>

Others are as follows.
Wave amplitude $A_i = 0.5$ m; Water depth $h = 20$ m; Young's modulus $E = 10^7$ N/m²; Mass moment of inertia $= 10^{-2}$ m⁴; Draft $d = 2$ m.

![Figure 7: Control forces for three conditions.](image)

Firstly, traditional structural design depends on structural strength and the capability to dissipate energy arising from dynamic forces such as machine loading, wind forces, and earthquakes. The control effects can be improved by the active control forces. Secondly, stability analysis and systematic design are certainly among the most important issues for active control systems. The proposed T-S fuzzy model and PDC fuzzy control (based on Lyapunov functions) can easily ensure stabilization of TLP structures. The simulation results demonstrate the TLP system to be stable. The trajectory of the initial conditions, starting from nonzero initial states, is kept within certain bounds under wave excitation.

5. Conclusion

In order to exploit rich new sources of oil and gas lying beneath the ocean, we must be ready to delve into very deep waters. Many promising fields are situated under water
that is in the 4000–7000 ft depth range. A favored platform design in that environment is the TLP system, where the hull is connected to the seabed by strings of tendon pipes. Since tendon length as well as water pressure increases in proportion to water depth, it is essential to produce a stable structure that the tendon pipes be highly resistant to collapse, especially in deep water environments. We present here a new control force concept for the stabilization of TLP systems, an alternative to the tether drag effects used previously. The results of the present study show that this new controller improves the limitations of steel performance for the maximum water depth attainable with the old TLP system. The dependence of wave reflection, transmission, as well as structure surge motion on incident wave conditions and structure properties has been demonstrated. The results show that the response of the floating structure is influenced by the its mass (as demonstrated by the different surge motion profiles). In particular, the response of a floating structure reaches the largest level of displacement during resonance. The resonant phenomenon takes place when the dimensionless wave frequency moves to lower values, approximately in the 0.5–1.0 range. The response to the wave-structure interaction results in platform vibration, but this can be slowed down and stabilized by means of the control force.

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