Research Article

Model Validation Using Coordinate Distance with Performance Sensitivity

Jiann-Shiun Lew
Center of Excellence in Information Systems, Tennessee State University, Nashville, TN 37209, USA
Correspondence should be addressed to Jiann-Shiun Lew, lew@coe.tsuniv.edu
Received 21 February 2008; Accepted 20 July 2008

This paper presents an innovative approach to model validation for a structure with significant parameter variations. Model uncertainty of the structural dynamics is quantified with the use of a singular value decomposition technique to extract the principal components of parameter change, and an interval model is generated to represent the system with parameter uncertainty. The coordinate vector, corresponding to the identified principal directions, of the validation system is computed. The coordinate distance between the validation system and the identified interval model is used as a metric for model validation. A beam structure with an attached subsystem, which has significant parameter uncertainty, is used to demonstrate the proposed approach.

1. Introduction

Model validation of structural dynamics is of great interest to both government and industry [1]. Recently, a model validation workshop [2, 3] was organized by Sandia National Laboratories to address the problem of certification of structures under various forms of uncertainty. Following their formulation, an integrated system consisting of a beam structure and an attached subsystem, shown in Figure 1, is the test structure used for study. In this model the physical elements of the attached three degrees of freedom subsystem are the only ones exhibiting significant parameter variations, all other parameters are known. The substructure, along with its nonlinear connection, is considered for calibration, and data are provided as a basis for the calibration of the substructure model [2].

In the process of certifying structures for use in harsh dynamic environments, it is often required that not only the main structure be capable of withstanding the loads but also all the attached substructures. To ensure survivability of all the substructures, Sandia in [2] has chosen a performance metric in terms of the maximum acceleration magnitude of mass 3, top of the substructure, under a shock force at position $x_8$. For this study, the uncertain
parameters are the identified modal parameters (frequency, damping, and mode shape) of subsystem, 15 parameters total.

This paper presents a model validation methodology based on an interval modeling technique for the structural dynamics problem proposed by Sandia [2]. A singular value decomposition technique [4] is applied to extract the principal components of parameter change, where the sensitivity of performance is included in the SVD process. From this process, an interval model is generated and each interval corresponds to one identified bounded uncertainty parameter with its associated principal direction. This interval modeling technique can precisely quantify the uncertainty of a system with significant parameter uncertainty [4]. The coordinate vector, corresponding to the identified principal directions, of the validation system can be computed. The coordinate distance between the validation system and the identified interval model is used as the metric for model validation [5].

2. Model validation

In the model validation process, first an interval modeling technique, given in the appendix, is applied for uncertainty quantification. The data used for model uncertainty quantification are based on the identified modal parameters from 60 virtual experiments [2], generated from 20 identical systems selected from a virtual pool and three levels of random excitation applied at mass 2. The modal parameter vector of the subsystem is defined as

$$p = [\omega_1 \omega_2 \omega_3 \xi_1 \xi_2 \xi_3 \phi_{11} \phi_{21} \phi_{31} \phi_{12} \cdots \phi_{33}]^T,$$

where $\omega_i$ is the $i$th natural frequency, $\xi_i$ is the $i$th damping ratio, and $\phi_{ji}$ is the $j$th component of the $i$th mode shape. The interval modeling technique in the appendix is applied to generate an interval model as

$$P = \left\{ p \mid p = p_0 + \sum_{j=1}^{15} \alpha_j q_j, \alpha_j \in \left[ \alpha^-_j, \alpha^+_j \right] \right\} ,$$

where $p_0$ is the nominal parameter vector, and $\alpha_j$ is the $j$th identified bounded uncertainty parameter corresponding to the basis vector $q_j$. The coordinate vector of any validation system with parameter vector $p^v$ can be computed as

$$\beta^v = U^{-1} \Delta p^v ,$$
Figure 2: Modal parameters of subsystems: (a) natural frequency (rad/sec) of 1st mode, (b) damping ratio of 3rd mode, (c) 2nd mode shape coefficient of 1st mode. (circels) 60 calibration systems; (asterisks) 60 validation systems.

with

$$\Delta p^v = p^v - p_0$$

$$U = \begin{bmatrix} q_1 & \cdots & q_{15} \end{bmatrix},$$

(2.4)

where \(U\) is the basis matrix. The coordinate distance between a validation system and the interval model is defined as

$$d^v = \min \left\{ \sqrt{[\beta^v - \beta(p)]^T [\beta^v - \beta(p)]}, \ p \in P \right\},$$

(2.5)

where \(\beta(p)\) is the coordinate vector of the subsystem with parameter vector \(p\). This distance represents a metric of performance deviation between a validation system and the identified interval model since the weighting of performance sensitivity is included in SVD process [4, 5].

3. Discussion of results

There are 60 sets of identified modal parameters used for model validation [2], generated from 20 identical systems selected from a virtual pool with three levels of shock input at mass 1. Figure 2 shows three modal parameters of 60 calibration systems and 60 validation systems as functions of the first uncertainty parameter \(\alpha_1\). Variations in the natural frequencies are significant, around 100%, and increase linearly as the first uncertainty parameter \(\alpha_1\) increases. Natural frequencies of calibration systems and validation systems share same variation characteristics. Damping and mode shape coefficients of validation systems show bias from those of calibration systems. For example, the mean value of \(\xi_3\) of the validation systems
Figure 3: Coordinates and parameter bounds of uncertainty parameters: (a) 2nd uncertainty parameter, (b) 3rd uncertainty parameter, (c) 4th uncertainty parameter. (circles) 60 calibration systems; (asterisks) 60 validation systems; — parameter bounds of interval model.

is around 30% lower than that of the calibration systems when \( \alpha_1 \) is 0.1. For the second mode shape coefficient of the first mode in Figure 2, the mean value for the validation systems is always around 5% lower than that of the calibration systems. Figure 3 shows the uncertainty parameters \( \alpha_2-\alpha_4 \) and the identified interval bounds as functions of the first uncertainty parameter \( \alpha_1 \). The third interval length, normalized to the first interval length (i.e., \( \alpha_1 = 1 \)), drops to less than 10% (see Figure 3) of the first interval length [4]. The model uncertainty is dominated by the first uncertainty parameter \( \alpha_1 \). Natural frequency variations are the dominant uncertainty corresponding to variations in \( \alpha_1 \). In contrast to frequency variations, damping and mode shape variations behave more like random variables, and they correspond to secondary uncertainties [4]. All \( \alpha_2 \) and \( \alpha_3 \) of validation systems are inside the bounds or close to the boundary of the identified interval model. All \( \alpha_4 \) of validation systems are outside the bounds of the interval model, and this bias is mainly contributed from the bias of mode shape and damping. Figure 4 shows the coordinate distance of 60 validation systems from interval model. The distance is mainly due to the bias of \( \alpha_4 \).

Figure 5 shows the performance sensitivity to the identified uncertainty parameters \( \alpha_i \). The sensitivity of performance to the \( j \)th uncertainty parameters \( \alpha_j \) of the \( i \)th chosen subsystem \( p_i \) is defined as

\[
S_{ij}^a = \frac{1}{a(p')} \left| \frac{\partial a(p')}{\partial \alpha_j} \right|, \quad i = 1, \ldots, n_s, \tag{3.1}
\]

where \( a(p') \) is the maximum acceleration magnitude of the integrated system with subsystem parameter vector \( p' \), and \( n_s \) is the number of parameter vectors. This sensitivity represents a
Figure 4: Coordinate distance of 60 validation systems from interval model: (circles) distance from interval model; (asterisks) distance contributed from $\alpha_4$ bias.

Figure 5: Sensitivity of performance to identified uncertainty parameters $\alpha_j$.

Percentage change. The average sensitivity corresponding to the $j$th uncertainty parameters $\alpha_j$ is defined as

$$s_{\alpha_j} = \frac{1}{n_s} \sum_{i=1}^{n_s} |s_{\alpha_j}^i|.$$  

Figure 5 shows that this sensitivity is between 21% and 69%, corresponding to the original maximum acceleration magnitude, and the sensitivity to $\alpha_4$ is 39% of the maximum acceleration. Coordinate distance of all the validation systems is between 0.03 and 0.07. This means that the maximum acceleration deviation between the validation system and a system in interval model is insignificant (around 1% to 3%), based on the sensitivity in Figure 5. All the validation systems are acceptable, based on the coordinate distance corresponding to performance index of maximum acceleration.

Figure 6 shows the maximum acceleration of the integrated systems with the identified interval model, 60 calibration systems, and 60 validation systems when an impulse force is applied at $x_8$ position. The results show that the identified interval model well represents and covers 60 calibration systems. The maximum acceleration of all the validation systems is inside the envelope or close to the boundary of the interval model. As expected, the validation systems are acceptable, based on the coordinate distance results shown in Figure 4. This
coordinate distance represents a metric of the maximum acceleration deviation (percentage difference) between a validation system and the identified interval model.

4. Concluding remarks

This paper presents a novel approach for model validation of a system with an attached sub-system that is exhibiting significant parameter uncertainty. An interval modeling technique is applied for uncertainty quantification with the performance sensitivity weighting in SVD process. The coordinate distance, between the validation system and the identified interval model, is defined as a metric for model validation. This distance represents a metric of the possible performance deviation of the validation system from a system in interval model. The results show that all the validation systems provided by Sandia are acceptable, based on this distance metric. This demonstrates an efficient tool for model validation, based on the interval model analysis. The proposed technique in this paper can be extended to probability framework.

Appendix

Model uncertainty quantification

The sensitivity of performance index \( a \), such as maximum acceleration magnitude, to the \( j \)th component of the \( i \)th chosen subsystem \( p^i \) is defined as

\[
s_{ij} = \frac{1}{a(p^i)} \left| \frac{\partial a(p^i)}{\partial p^ij} \right| \sigma_j, \quad i = 1, \ldots, n_s, \tag{A.1}
\]

where \( p^ij \) is the \( j \)th component of parameter vector \( p^i \), and \( \sigma_j \) is the standard deviation of the \( j \)th vector component. This sensitivity represents a percentage change including the factor \( \sigma_j \) to account for the size of the parameter variation. The average sensitivity corresponding to the \( j \)th vector component is defined as

\[
s_j = \frac{1}{n_s} \sum_{i=1}^{n_s} |s_{ij}|. \tag{A.2}
\]
Jiann-Shiun Lew

To quantify the parameter uncertainty, an uncertainty matrix is defined as

$$\Delta P = [\Delta p_1 \ \Delta p_2 \ \cdots \ \Delta p_n], \quad \text{(A.3)}$$

with

$$\Delta p_j = p_j - p_0, \quad j = 1, \ldots, n, \quad p_0 = \frac{1}{n} \sum_{j=1}^{n} p_j, \quad \text{(A.4)}$$

where \(p_j\) is the \(j\)th identified parameter vector, and \(p_0\) is the nominal parameter vector, which is computed as the average from \(n = 60\) experiments.

A singular value decomposition (SVD) technique \([4]\) is used to generate an optimal linear interval model. This SVD process involves the following computational steps.

1. Compute an initial weighting matrix as

$$\Delta P^1 = W_1^{-1} \Delta P, \quad \text{(A.5)}$$

where \(W_1\) is a diagonal matrix with its \(j\)th diagonal element as the standard deviation \(\sigma_j\).

2. Compute the weighting matrix including sensitivity as

$$\Delta P^W = W_2 \Delta P^1, \quad \text{(A.6)}$$

where \(W_2\) is a diagonal matrix with its \(j\)th diagonal element \(s_j\).

3. Use SVD to compute the basis matrix \(U^W\) for \(\Delta P^W\),

$$\Delta P^W = U^W S V^T, \quad S = \text{diag} [d_1 \ \cdots \ \ d_{15}], \quad \text{(A.7)}$$

4. Compute the basis matrix \(U\) for \(\Delta P\),

$$U = W_1 W_2^{-1} U^W, \quad U = [q_1 \ \cdots \ \ q_{15}], \quad \text{(A.8)}$$

The singular values \(d_i\) are in descending order, this leads to a descending order of perturbation distribution in \(q_j\).

5. Compute the coordinate vector of \(\Delta p_i\) corresponding to the basis vectors \(q_j\),

$$\beta_i = U^{-1} \Delta p_i, \quad \text{(A.9)}$$

6. Represent each parameter vector as

$$p_i = p_0 + \sum_{l=1}^{15} \beta_i(l) q_l, \quad \text{(A.10)}$$

where \(\beta_i(l)\) is the \(l\)th element of the coordinate vector \(\beta_i\).

7. Compute the parameter bounds as

$$\alpha_i^+ = \max \{\beta_1(j), \beta_2(j), \ldots, \beta_n(j)\},$$

$$\alpha_i^- = \min \{\beta_1(j), \beta_2(j), \ldots, \beta_n(j)\}, \quad \text{(A.11)}$$

All the basis vectors, coordinates, and parameter bounds are normalized to the first interval length \([6]\).
References


