Research Article

The Hypergroupoid Semigroups as Generalizations of the Groupoid Semigroups

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We introduce the notion of hypergroupoids \((H\text{Bin}(X), \square)\), and show that \((H\text{Bin}(X), \square)\) is a super-semigroup of the semigroup \((\text{Bin}(X), \square)\) via the identification \(x \leftrightarrow \{x\}\). We prove that \((H\text{Bin}^*(X), \oplus, [0])\) is a BCK-algebra, and obtain several properties of \((H\text{Bin}^*(X), \square)\).

1. Introduction

The notion of the semigroup \((\text{Bin}(X), \square)\) was introduced by Kim and Neggers [1]. Fayoumi [2] introduced the notion of the center \(Z\text{Bin}(X)\) in the semigroup \(\text{Bin}(X)\) of all binary systems on a set \(X\), and showed that if \((X, \cdot) \in Z\text{Bin}(X)\), then \(x \neq y\) implies \(\{x, y\} = \{x \cdot y, y \cdot x\}\). Moreover, she showed that a groupoid \((X, \cdot) \in Z\text{Bin}(X)\) if and only if it is a locally zero groupoid. Y. Imai and K. Iseki introduced two classes of abstract algebras: BCK-algebras and BCI-algebras [3, 4]. Neggers and Kim introduced the notion of \(d\)-algebras which is another useful generalization of BCK-algebras, and then investigated several relations between \(d\)-algebras and BCK-algebras as well as several other relations between \(d\)-algebras and oriented digraphs [5]. The present authors [6] defined several special varieties of \(d\)-algebras, such as strong \(d\)-algebras, (weakly) selective \(d\)-algebras, and pre-\(d\)-algebras, discussed the associative groupoid product \((X; \square) = (X; \ast)\square(X; \circ)\), where \(x \square y = (x \ast y) \circ (y \ast x)\). They showed that the squared algebra \((X; \square, 0)\) of a pre-\(d\)-algebra \((X; \ast, 0)\) is a strong \(d\)-algebra if and only if \((X; \ast, 0)\) is strong.

Zhan et al. [7] defined the \(T\)-fuzzy \(n\)-ary sub-hypergroups by using a norm \(T\) and obtained some related properties. Zhan, and Liu [8] introduced the notion of \(f\)-derivation of

In this paper we introduce the notion of hypergroupoids $(H\text{Bin}(X), \Box)$, and show that $(H\text{Bin}(X), \Box)$ is a super-semigroup of the semigroup $(\text{Bin}(X), \Box)$ via the identification $x \mapsto \{x\}$. We prove that $(H\text{Bin}^*(X), \ominus, \{\emptyset\})$ is a BCK-algebra, and obtain several properties of $(H\text{Bin}^*(X), \Box)$.

### 2. Preliminaries

Given a nonempty set $X$, we let $\text{Bin}(X)$ the collection of all groupoids $(X, *)$, where $*: X \times X \rightarrow X$ is a map and where $*(x, y)$ is written in the usual product form. Given elements $(X, *)$ and $(X, \bullet)$ of $\text{Bin}(X)$, define a product “$\Box$” on these groupoids as follows:

$$(X, *) \Box (X, \bullet) = (X, \Box),$$

where

$$x \Box y = (x \ast y) \bullet (y \ast x),$$

for any $x, y \in X$. Using the notion, H. S. Kim and J. Neggers showed the following theorem.

**Theorem 2.1** (see [1]). $(\text{Bin}(X), \Box)$ is a semigroup, that is, the operation “$\Box$” as defined in general is associative. Furthermore, the left zero semigroup is an identity for this operation.

### 3. Hypergroupoid Semigroups

Instead of a groupoid $(X, *)$ on $X$, we may also consider a *hypergroupoid* $(X, \varphi)$ on $X$, where $\varphi: X \times X \rightarrow P^*(X)$ is a *hyperproduct* with $P^*(X)$, the set of all non-empty subsets of $X$. We denote the set of all hypergroupoids $(X, \varphi)$ on $X$ by $H\text{Bin}(X)$, that is,

$$H\text{Bin}(X) := \{(X, \varphi) \mid \text{a hypergroupoid on } X\}.$$  

The product “$\Box$” discussed in $\text{Bin}(X)$ can be generalized in $H\text{Bin}(X)$ as follows: given $(X, \varphi), (X, \psi) \in H\text{Bin}(X)$, for any $x, y \in X$,

$$xy := (x\varphi y)\psi(y\psi x).$$

If we identify $x \in X$ with $\{x\} \in P^*(X)$, then we have an inclusion: $X \subseteq P^*(X)$ and thus for $\varphi(x, y) = x\varphi y \in P^*(X)$, we have $x\varphi y \subseteq X$ and hence also $x\varphi y \subseteq P^*(X)$ via this identification.

If $A, B \subseteq X$, then for the groupoid $(X, \ast) \in \text{Bin}(X)$, we have

$$A \ast B := \{a \ast b \mid a \in A, \ b \in B\},$$

where
hence \(|a| \ast \{b\} = \{ab\} \ast \{b\}\) in a natural way. Similarly, given a hypergroupoid \((X, \varphi) \in HBin(X)\), \(A \varphi B\) is defined by \(A \varphi B = \cup \{ x \varphi y \mid x \in A, \ y \in B \}\).

Given hypergroupoids \((X, \varphi), (X, \psi)\), we let \((X, \theta) := (X, \varphi) \Box (X, \psi)\). Then, for any \(x, y \in X\), we have

\[
x \theta y = (x \varphi y) \psi (y \varphi x)
\]

\[
= \cup \{ a \varphi b \mid a \in x \varphi y, \ b \in y \varphi x \}. \tag{3.4}
\]

Suppose that \((X, \ast)\) and \((X, \circledast)\) are groupoids and that we determine the following:

\[
x \theta y = (x \ast y) \circledast (y \ast x)
\]

\[
= \cup \{ a \circledast b \mid a \in \{x \ast y\}, \ b \in \{y \ast x\} \}
\]

\[
= \{ (x \ast y) \circledast (y \ast x) \}
\]

\[
= \{ x \Box y \} = x \Box y, \tag{3.5}
\]

via the identification \(x \leftrightarrow \{x\}\). Hence \((X, \ast) \Box (X, \circledast)\) is the same as a product of groupoids or as a product of hypergroupoids.

It can be shown that \((\text{Bin}(X), \Box) \rightarrow (H\text{Bin}(X), \Box)\) is an injection (an into homomorphism) via the identification \(x \leftrightarrow \{x\}\) and the associated identification \(x \theta y = \{x \Box y\} = x \Box y\).

Example 3.1. Let \(X := \mathbb{R}^2\) and for any \(x, y \in X\), let \(x \varphi y\) denote the undirected line segment connecting \(x\) with \(y\). Then \(x \varphi x = \{x\}\) and \(x \varphi y = y \varphi x\). Let \((X, \theta) := (X, \varphi) \Box (X, \varphi)\). Then \(x \theta y = \cup \{ a \varphi b \mid a \in x \varphi y, \ b \in y \varphi x \}\) for any \(x, y \in X\). Since \(x \varphi y = y \varphi x\), \(a \varphi b \subseteq x \theta y\) for any \(a, b \in x \varphi y\). Since \(x, y \in x \varphi y\), \(x \varphi y \subseteq x \theta y\). We claim that \(x \theta y \subseteq x \varphi y\). If \(a \in x \theta y\), then \(a \in a \varphi b\) for some \(a \in x \varphi y\) and \(b \in y \varphi x\). Since \(x \varphi y = y \varphi x\), \(a \in a \varphi b\) for some \(a, b \in x \varphi y\), which shows that \(a \in x \varphi y\). This proves that \((X, \varphi) = (X, \theta) = (X, \varphi) \Box (X, \varphi)\), that is, \((X, \varphi)\) is an idempotent hypergroupoid in \((H\text{Bin}(X), \Box)\).

Theorem 3.2. \((H\text{Bin}(X), \Box)\) is a supersemigroup of the semigroup \((\text{Bin}(X), \Box)\) via the identification \(x \leftrightarrow \{x\}\).

Proof. Suppose that \((X, \varphi), (X, \psi)\) and \((X, \omega)\) are hypergroupoids and let \((X, a) := (X, \varphi) \Box (X, \omega)\) and \((X, b) := (X, \varphi) \Box (X, \psi)\). Then for any \(x, y \in X\), we have \(x \varphi y = (x \varphi y) \omega (y \varphi x)\) and \(x \theta y = (x \varphi y) \psi (y \varphi x)\). Let \((X, \theta) := [(X, \varphi) \Box (X, \psi)] \Box (X, \omega)\). Then \((X, \theta) = (X, b) \Box (X, \omega)\) and hence we obtain the following

\[
x \theta y = (x \varphi y) \omega (y \varphi x)
\]

\[
= [(x \varphi y) \varphi (y \varphi x)] \varphi [(y \varphi x) \varphi (x \varphi y)]. \tag{3.6}
\]
If we let \((X, \mu) := (X, \varphi) \Box (X, \psi) W(X, \omega)\), then \((X, \mu) = (X, \varphi) \Box (X, \alpha)\) and hence \(x\mu y = (x\varphi y)\alpha (y\varphi x)\) for any \(x, y \in X\). Let \(p := x\varphi y, q := y\varphi x\). Then

\[
x\mu y = p\alpha q
\]

\[
= (p\varphi q)\omega (q\varphi p)
\]

\[
= [(x\varphi y)\varphi (y\varphi x)]\omega [(y\varphi x)\varphi (x\varphi y)].
\]

This proves that \((X, \theta) = (X, \mu)\), that is, \((H\text{Bin}(X), \Box)\) is a semigroup.

\[\square\]

**Proposition 3.3.** The left-zero-semigroup \((X, \ast)\), that is, \(x \ast y = x\) for any \(x, y \in X\), is an identity of the semigroup \((H\text{Bin}(X), \Box)\).

**Proof.** Let \((X, \ast)\) be a left-zero-semigroup. Then \((X, \ast) \in \text{Bin}(X)\). By the identification \(x \leftrightarrow \{x\}\), we have \((X, \ast) \in (H\text{Bin}(X), \Box)\). Given \((X, \nu) \in H\text{Bin}(X)\), let \((X, \theta) := (X, \ast) \Box (X, \nu)\). Then for any \(x, y \in X\), we have

\[
x\theta y = (x \ast y) \nu (y \ast x)
\]

\[
= \{x\} \nu \{y\}
\]

\[
= \cup \{a \nu b \mid a \in \{x\}, b \in \{y\}\}
\]

\[
= x\nu y,
\]

that is, \((X, \theta) = (X, \nu)\). This proves that \((X, \ast)\) is a left identity on \(H\text{Bin}(X)\).

Similarly, if we let \((X, \theta) = (X, \nu) \Box (X, \ast)\), then for any \(x, y \in X\),

\[
x\theta y = (x\nu y) \ast (y\nu x)
\]

\[
= \{a \ast b \mid a \in x\nu y, b \in y\nu x\}
\]

\[
= \{a \mid a \in x\nu y\}
\]

\[
= x\nu y,
\]

that is, \((X, \theta) = (X, \nu)\). This proves that \((X, \ast)\) is a right identity on \(H\text{Bin}(X)\).

\[\square\]

Given an element \((X, \varphi) \in H\text{Bin}(X), x\varphi y \in \mathcal{P}^*(X)\), that is, \(\emptyset \neq x\varphi y \subseteq X\). We extend \((X, \varphi)\) to \((\mathcal{P}^*(X), \bar{\varphi})\) as

\[
\bar{\varphi} : \mathcal{P}^*(X) \times \mathcal{P}^*(X) \rightarrow \mathcal{P}^*(\mathcal{P}^*(X))
\]

by \(\bar{\varphi}(A, B) := A\bar{\varphi}B\), where \(A\bar{\varphi}B = \cup \{a\varphi b \mid a \in A, b \in B\}\). In particular,

\[
\{x\} \bar{\varphi} \{y\} = \cup \{a\varphi b \mid a \in \{x\}, b \in \{y\}\}
\]

\[
= x\varphi y.
\]
This produces a mapping $\pi : H\text{Bin}(X) \rightarrow \text{Bin}P^*(X)$. Let $(X, \varnothing) := (X, \varnothing) \varnothing (X, \varnothing)$. Then $x\varnothing y = \cup \{a\varnothing b \mid a \in x\varnothing y, b \in y\varnothing x\}$ for any $x, y \in X$. Since $x\varnothing y, y\varnothing x \in P^*(X)$, we have

$$
(x\varnothing y)\varnothing (y\varnothing x) = \cup \{a\varnothing b \mid a \in x\varnothing y, b \in y\varnothing x\}
$$

(3.12)

Since $x\varnothing y = \{x\} \varnothing \{y\}$ via the identification $x \leftrightarrow \{x\}$, we obtain

$$
x\varnothing y = (x\varnothing y)\varnothing (y\varnothing x)
= (\{x\} \varnothing \{y\}) \varnothing (\{y\} \varnothing \{x\})
= x\varnothing y,
$$

(3.13)

where $(P^*(X), \varnothing) = (P^*(X), \varnothing) \varnothing (P^*(X), \varnothing)$ in $(\text{Bin}P^*(X), \varnothing)$. We claim that $\pi$ is a homomorphism. In fact, $\pi((X, \varnothing) \varnothing (X, \varnothing)) = \pi((X, \varnothing)) = (P^*(X), \varnothing) = (P^*(X), \varnothing) \varnothing (P^*(X), \varnothing) = \pi((X, \varnothing)) \varnothing \pi((X, \varnothing))$.

Given $H\text{Bin}(X)$, we may order it according to the rule

$$(X, \varnothing) \leq (X, \varnothing) \iff x\varnothing y \subseteq x\varnothing y, \ \forall x, y \in X.
$$

(3.14)

We define a mapping $[\varnothing] : X \times X \rightarrow P(X)$ by $[\varnothing](x, y) := \varnothing$ for all $x, y \in X$. If we let $H\text{Bin}^*(X) := H\text{Bin}(X) \cup \{(X, [\varnothing])\}$, then $(X, [\varnothing])$ is the minimal element of $(H\text{Bin}^*(X), \leq)$.

**Proposition 3.4.** Let $(X, \varnothing) \in H\text{Bin}(X)$ and $(X, \bullet) \in \text{Bin}(X)$. If $(X, \varnothing) \leq (X, \bullet)$, then $(X, \varnothing) = (X, \bullet)$.

**Proof.** If $(X, \varnothing) \leq (X, \bullet)$, then $\varnothing \neq x\varnothing y \subseteq \{x \bullet y\}$ for any $x, y \in X$. It follows that $x\varnothing y = \{x \bullet y\} = x \bullet y$, proving that $(X, \varnothing) = (X, \bullet)$. \qed

**Proposition 3.5.** Let $(X, \bullet), (X, \bullet) \in \text{Bin}(X)$. If $(X, \bullet) \leq (X, \bullet)$, then $(X, \bullet) = (X, \bullet)$, that is, Bin$(X)$ is an antichain in $(H\text{Bin}^*(X), \leq)$.

**Proof.** If $(X, \bullet) \leq (X, \bullet)$, then $\{x \bullet y\} \subseteq \{x \bullet y\}$ for any $x, y \in X$. It follows that $x \bullet y = x \bullet y$ for any $x, y \in X$, proving that $(X, \bullet) = (X, \bullet)$. \qed

### 4. BCK-Algebras on $H\text{Bin}^*(X)$

In this section we discuss BCK-algebras on $H\text{Bin}^*(X)$ by introducing a binary operation as follows: given hypergroupoids $(X, \varnothing), (X, \varnothing) \in H\text{Bin}^*(X)$, we define a binary operation “$\ominus$” by

$$(X, \varnothing) \ominus (X, \varnothing) := (X, \varnothing \ominus \varnothing),
$$

(4.1)

where $x(\varnothing \ominus \varnothing)y := x\varnothing y \setminus x\varnothing y$ for any $x, y \in X$. 

Theorem 4.1. \((HBin^*(X), \ominus, [\emptyset])\) is a BCK-algebra.

Proof. For any \((X, \varphi) \in HBin^*(X)\), since \(x[\emptyset]y \setminus x\varphi y = \emptyset\) for any \(x, y \in X\), we have \((X, [\emptyset]) \ominus (X, \varphi) = (X, [\emptyset])\).

Given \((X, \varphi) \in HBin^*(X)\), since \(x\varphi y \setminus x\varphi y = \emptyset\) for any \(x, y \in X\), we have \((X, \varphi) \ominus (X, \varphi) = (X, [\emptyset])\).

Assume that \((X, \varphi) \ominus (X, \varphi) = (X, [\emptyset]) = (X, [\emptyset]) \ominus (X, \varphi). Then \(x\varphi y \setminus x\varphi y = \emptyset\) for any \(x, y \in X\), which shows that \(x\varphi y = x\varphi y\) for any \(x, y \in X\), that is, \((X, \varphi) = (X, \varphi)\).

Given \((X, \varphi), (X, \psi), (X, \delta) \in HBin^*(X)\), since \([x\varphi y \setminus x\varphi y] \setminus x\varphi y = \emptyset\) for any \(x, y \in X\), we obtain \([(X, \varphi) \ominus (X, \varphi) (X, \psi)] \ominus (X, \psi) = (X, [\emptyset])\).

Given \((X, \varphi), (X, \psi), (X, \delta) \in HBin^*(X)\), since \([(x\varphi y \setminus x\varphi y) \setminus (x\varphi y \setminus x\varphi y) = \emptyset\) for any \(x, y \in X\), we obtain \([(X, \varphi) \ominus (X, \varphi) (X, \psi)] \ominus (X, \delta) = (X, [\emptyset])\).

This proves the theorem.

\[\square\]

5. Several Properties on \(HBin(X)\)

In this section, we discuss some properties on \(HBin(X)\).

Proposition 5.1. The product \(\square\) is order-preserving, that is, if \((X, \varphi) \leq (X, \xi), (X, \psi) \leq (X, \omega)\), then \((X, \varphi) \square (X, \varphi) \leq (X, \xi) \square (X, \omega)\).

Proof. Let \((X, \varphi) \leq (X, \xi), (X, \psi) \leq (X, \omega)\) in \(HBin(X)\). If we let \((X, \theta) := (X, \varphi) \square (X, \varphi)\) and \((X, \rho) := (X, \xi) \square (X, \omega)\), then for any \(x, y \in X\),

\[
x\theta y = (x\varphi y)\varphi(y\varphi x) \\
\leq (x\xi y)\varphi(y\varphi x) \\
\leq (x\xi y)\omega(y\varphi x) \\
= x\psi y,
\]

proving that \((X, \theta) \leq (X, \rho)\).

\[\square\]

We define a mapping \([X] : X \times X \to P(X)\) by \([X](x, y) := X\) for all \(x, y \in X\). Then \((X, [X])\) is the maximal element of \((HBin^*(X), \leq)\). Given \((X, \varphi) \in HBin(X)\), if we let \((X, \theta) := (X, [X]) \square (X, \varphi)\), then \(x\theta y = (x[X] y \varphi(y[X] x) = X\varphi X = \cup \{a \varphi b \mid a, b \in X\}\) for any \(x, y \in X\).

Proposition 5.2. If \((X, \varphi) \in HBin(X)\), then \((X, \varphi) \square (X, [X]) = (X, [X])\).

Proof. Let \((X, \theta) := (X, \varphi) \square (X, [X])\). Then, for any \(x, y \in X\), we have

\[
x\theta y = (x\varphi y)[X](y\varphi x) \\
= \cup \{a[X] b \mid a \in x\varphi y, b \in y\varphi x\} \tag{5.2}
\]

\[= X,\]

proving that \((X, \theta) = (X, [X])\).

\[\square\]
Given \((X, \varphi) \in H\text{Bin}^*(X)\), we define a hypergroupoid \((X, \varphi^C)\) by \(x\varphi^Cy := X \setminus x\varphi y\), for any \(x, y \in X\). We call it the **complementary hypergroupoid** of \((X, \varphi^C)\).

For example, if \((X, \cdot, e)\) is a group, then \(x^{-C}y = X \setminus \{x \cdot y\}\), where \(x, y \in X\). It follows that \(x^{-C}e = e^{-C}x = X \setminus \{x\}\) and \(x^{-C}x^{-1} = x^{-1}^{-C}x = X \setminus \{e\}\) for any \(x \in X\).

A hypergroupoid \((X, \varphi)\) is said to be a **complementary d-algebra** if there exists \(0 \in X\) such that (i) \(x\varphi x = X \setminus \{0\}\); (ii) \(0\varphi x = X \setminus \{0\}\); (iii) \(x\varphi y = y\varphi x = X \setminus \{x\}\) implies \(x = y\), for any \(x, y \in X\).

The following proposition can be easily seen.

**Proposition 5.3.** Given \((X, \varphi) \in H\text{Bin}^*(X)\), \((X, \varphi)\) is a d-algebra if and only if \((X, \varphi^C)\) is a complementary d-algebra.

**Example 5.4.** Let \(X := \mathbb{R}\) be the set of all real numbers and \(f : X \to X\) be a mapping. Define a map \(\varphi_f : X \times X \to \mathbb{R}^+\) by \(\varphi_f(x, y) := [x - |f(y)|, x + |f(y)|]\). Then \((X, \varphi_f)\) be a hypergroupoid for which \(x\varphi_f y = [x - |f(y)|, x + |f(y)|]\) has a midpoint \(x\) where \(x, y \in X\).

In particular, let \(f(x) := x^2\) for any \(x \in X\) and let \((X, \varphi) := (X, \varphi_f)\Box(X, \varphi_f)\). Then \(x\varphi y = (x\varphi y)\varphi_f (y\varphi x) = \{a \varphi b \mid a \in [x - |f(y)|, x + |f(y)|], b \in [y - |f(x)|, y + |f(x)|]\} = \{a - b^2, a + b^2 \mid a \in [x - y^2, x + y^2], b \in [y - x^2, y + x^2]\} = [x - 2y(y + x^2) - y^4, x + 2y(y + x^2) + x^4]\), an interval of length \(y^2 + (y + x^2)^2 \geq 0\), where \(x = y = 0\) implies \(0\theta 0 = [0, 0] = \{0\}\), corresponding to 0 in the identification.

A hypergroupoid \((X, \varphi)\) is said to be **left inclusive** if \(x \in x\varphi y\) for any \(x, y \in X\).

Note that the only left inclusive hypergroupoid which is a groupoid is the left-zero-semigroup. In fact, let \((X, \ast)\) be a left inclusive hypergroupoid which is a groupoid. Then \(x \in \{x \ast y\}\) for any \(x, y \in X\). It follows that \(x = x \ast y\) for any \(x, y \in X\), that is, \((X, \ast)\) is a left-zero-semigroup.

**Proposition 5.5.** The left inclusive hypergroupoids on \(X\) relative to the product “\(\Box\)” on \(H\text{Bin}(X)\) form a subsemigroup of \((H\text{Bin}(X), \Box)\).

**Proof.** Let \((X, \varphi), (X, \psi)\) be left inclusive hypergroupoids and let \((X, \theta) := (X, \varphi)\Box(X, \psi)\). Then \(x\varphi y = (x\varphi y)\varphi_f (y\varphi x) = \{a \varphi b \mid a \in x\varphi y, b \in y\varphi x\}\) for any \(x, y \in X\). Since \((X, \varphi)\) is left inclusive, \(x \in x\varphi y\), \(y \in y\varphi x\), and hence \(x\varphi y \subseteq x\varphi y \subseteq x\theta y\) for any \(x, y \in X\). Moreover, \((X, \varphi)\) is left inclusive implies that \(x \in x\varphi y\), which proves that \(x \in x\theta y\).

**Proposition 5.6.** Let \((X, \varphi) \leq (X, \psi)\) in \(H\text{Bin}(X)\). If \((X, \varphi)\) is left inclusive, then \((X, \psi)\) is also left inclusive.

**Proof.** Let \((X, \varphi) \leq (X, \psi)\). Then \(x\varphi y \subseteq x\psi y\) for any \(x, y \in X\). Since \((X, \varphi)\) is left inclusive, we have \(x \in x\varphi y \subseteq x\psi y\), proving the proposition.

**Proposition 5.6** means that the collection of all left inclusive hypergroupoids is a filter in the poset \((H\text{Bin}(X), \leq)\).

A hypergroupoid \((X, \varphi)\) is said to be **left-self-avoiding** if \(x \notin x\varphi y\) for any \(x, y \in X\).

**Proposition 5.7.** The complementary hypergroupoid \((X, \varphi^C)\) of a left inclusive hypergroupoid \((X, \varphi)\) is left-self-avoiding.
Proof. Let \((X, \varphi^C)\) be the complementary hypergroupoid of a left inclusive hypergroupoid \((X, \varphi)\). Then \(x \varphi^C y = X \setminus x \varphi y\) for any \(x, y \in X\). Since \((X, \varphi)\) is left inclusive, \(x \in x \varphi y\) for any \(x, y \in X\), and hence \(x \not\in x \varphi^C y\), proving the proposition. \(\square\)

**Proposition 5.8.** The complementary hypergroupoid \((X, \varphi^C)\) of a left-self-avoiding hypergroupoid \((X, \varphi)\) is left inclusive.

Proof. Straightforward. \(\square\)

**Proposition 5.9.** Let \((X, \theta) = (X, \varphi) \Box (X, \varphi)\) where \((X, \varphi)\) is left inclusive and \((X, \theta)\) is left-self-avoiding. Then \((X, \varphi)\) is left-self-avoiding.

Proof. Let \((X, \theta)\) be a left-self-avoiding hypergroupoid. Then \((X, \theta^C)\) is left inclusive by Proposition 5.8. It follows that \(x \in x \theta^C y = X \setminus \{a \varphi b \mid a \in x \varphi y, b \in y \varphi x\}\). This means that \(x \not\in a \varphi b\) for any \(a \in x \varphi y\) and \(b \in y \varphi x\). Hence \(x \not\in x \varphi y\), proving that \((X, \varphi)\) is left-self-avoiding. \(\square\)

### 6. Conclusion

In this paper we have introduced the notion of hypergroupoids as a generalization of groupoids in a manner analogous to the introduction of the notion of hypergroups as a generalization of the notion of groups. Since the semigroup \((\text{Bin}(X), \Box)\) can still benefit from more detailed investigation it follows that the same is even more true for \((H\text{Bin}(X), \Box)\). In the latter case one must rely on proper adaptations obtained from \((\text{Bin}(X), \Box)\) and certainly on results obtained from studies on hypergroupoids available in the literature \[7-10\] as a general plan for the organization of the subject, with parts to be completed as time and opportunity permits.

### References


