Research Article

On Fuzzy Corsini’s Hyperoperations

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Received 22 February 2012; Accepted 7 May 2012

Academic Editor: Said Abbasbandy

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We generalize the concept of C-hyperoperation and introduce the concept of F-C-hyperoperation. We list some basic properties of F-C-hyperoperation and the relationship between the concept of C-hyperoperation and the concept of F-C-hyperoperation. We also research F-C-hyperoperations associated with special fuzzy relations.

1. Introduction and Preliminaries

Hyperstructures and binary relations have been studied by many researchers, for instance, Chvalina [1, 2], Corsini and Leoreanu [3], Feng [4], Hort [5], Rosenberg [6], Spartalis [7], and so on.

A partial hypergroupoid $\langle H, \ast \rangle$ is a nonempty set $H$ with a function from $H \times H$ to the set of subsets of $H$.

A hypergroupoid is a nonempty set $H$, endowed with a hyperoperation, that is, a function from $H \times H$ to $P(H)$, the set of nonempty subsets of $H$.

If $A, B \in P(H) - \{\emptyset\}$, then we define $A \ast B = \cup\{a \ast b \mid a \in A, b \in B\}$, $x \ast B = \{x\} \ast B$ and $A \ast y = A \ast \{y\}$.

A Corsini’s hyperoperation was first introduced by Corsini [8] and studied by many researchers; for example, see [3, 8–15].

Definition 1.1 (see [8]). Let $\langle H, R \rangle$ be a a pair of sets where $H$ is a nonempty set and $R$ is a binary relation on $H$. Corsini’s hyperoperation (briefly, $C$-hyperoperation) $\ast_R$ associated with
2. Fuzzy Corsini’s Hyperoperation

In this section, we will generalize the concept of Corsini’s hyperoperation and introduce the fuzzy version of Corsini’s hyperoperation.

**Definition 2.1.** Let \((H, R)\) be a pair of sets where \(H\) is a non-empty set and \(R\) is a fuzzy relation on \(H\). We define a fuzzy hyperoperation \(\ast_R : H \times H \to F(H)\), for any \(x, y, z \in H\), as follows:

\[
(x \ast_R y)(z) = R(x, z) \land R(z, y).
\]
Table 1

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<td>R</td>
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<tr>
<td>a</td>
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<td>0.2</td>
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Table 2

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<thead>
<tr>
<th>*R</th>
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<tr>
<td>a</td>
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<td>0.1/a + 0.2/b</td>
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<tr>
<td>b</td>
<td>0.1/a + 0.3/b</td>
<td>0.2/a + 0.4/b</td>
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*R* is called a *fuzzy Corsini’s hyperoperation* (briefly, *F-C-hyperoperation*) associated with *R*. The fuzzy hyperstructure \( (H, *R) \) is called a partial F-C-hypergroupoid.

**Remark 2.2.** It is obvious that the concept of F-C-hyperoperation is a generalization of the concept of C-hyperoperation.

**Example 2.3.** Letting \( H = \{a, b\} \) be a non-empty set, *R* is a fuzzy relation on \( H \) as described in Table 1.

From the previous definition, by calculating, for example, \( (a *_{R} a)(a) = R(a, a) \land R(a, a) = 0.1 \land 0.1 = 0.1 \), \( R(a * b)(a) = R(a, a) \land R(a, b) = 0.1 \land 0.2 = 0.1 \), we can obtain Table 2 which is a partial F-C-hypergroupoid.

**Definition 2.4.** Supposing \( R, S \) are two fuzzy relations on a non-empty set \( H \), the composition of \( R \) and \( S \) is a fuzzy relation on \( H \) and is defined by \( (R \circ S)(x, y) = \bigvee_{z \in H} (R(x, z) \land S(z, y)) \), for all \( x, y \in H \).

**Proposition 2.5.** A partial F-C-hypergroupoid \( (H, *_{R}) \) is a F-C-hypergroupoid if and only if \( \text{supp}(R \circ R) = H \times H \), where \( \text{supp}(R \circ R) = \{(x, y) \mid (R \circ R)(x, y) \neq 0\} \).

**Proof.** Suppose that \( (H, *_{R}) \) is a hypergroupoid. For any \( x, y \in H \), there exists at least one \( z \in H \), such that \( (x *_{R} y)(z) \neq 0 \) holds.

So \( (R \circ R)(x, y) = \bigvee_{z \in H} (R(x, z) \land R(z, y)) \neq 0 \). Thus \( (x, y) \in \text{supp}(R \circ R) \). And we conclude that \( H \times H \subseteq \text{supp}(R \circ R) \).

\[ \text{supp}(R \circ R) \subseteq H \times H \text{ is obvious. And so } \text{supp}(R \circ R) = H \times H. \]

Conversely, if \( \text{supp}(R \circ R) = H \times H \), then for any \( x, y \in H \), \( (x, y) \in H \times H = \text{supp}(R \circ R) \).

So \( (R \circ R)(x, y) = \bigvee_{z \in H} (R(x, z) \land R(z, y)) \neq 0 \). That is, there exists at least one \( z \in H \) such that \( (x *_{R} y)(z) \neq 0 \) holds. And so \( (H, *_{R}) \) is a hypergroupoid.

Thus we complete the proof. \( \square \)

**Definition 2.6.** Letting \( H \) be a non-empty set, \( * \) is a fuzzy hyperoperation of \( H \), the hyperoperation \( *_{p} \) is defined by \( x *_{p} y = (x \ast y)_{p} \), for all \( x, y \in H \), \( p \in [0, 1] \). \( *_{p} \) is called the p-cut of \( * \).
Definition 2.7. Letting $R$ be a fuzzy relation on a non-empty set $H$, we define a binary relation $R_p$ on $H$, for all $p \in (0, 1]$, as follows:

$$xR_py \iff R(x, y) \geq p.$$  \hspace{1cm} (2.2)

$R_p$ is called the p-cut of the fuzzy relation $R$.

Proposition 2.8. Let $(H, \ast_R)$ be a partial F-C-hypergroupoid. Then $(\ast_R)_p$ is a C-hyperoperation associated with $R_p$, for all $0 < p \leq 1$.

Proof. For any $0 < p \leq 1$ and for any $x, y \in H$, we have

$$x(\ast_R)_p y = (x \ast_R y)_p = \{ z \in H \mid (x \ast_R y)(z) \geq p \} = \{ z \in H \mid R(x, z) \wedge R(z, y) \geq p \}$$

$$= \{ z \in H \mid R(x, z) \geq p, R(z, y) \geq p \} = \{ z \in H \mid xR_p z, zR_p y \}.$$  \hspace{1cm} (2.3)

From the definition of C-hyperoperation, we conclude that $(\ast_R)_p$ is a C-hyperoperation associated with $R_p$.

Thus we complete the proof. \hfill \Box

From the previous proposition and the construction of the F-C-hyperoperation, we can easily conclude that a fuzzy hyperoperation is a F-C-hyperoperation if and only if every p-cut of the F-C-hyperoperation is a C-hyperoperation. That is, consider the following.

Proposition 2.9. Let $H$ be a non-empty set and let $\ast$ be a fuzzy hyperoperation of $H$, then the fuzzy hyperoperation $\ast$ is an F-C-hyperoperation associated with a fuzzy relation $R$ on $H$ if and only if $\ast_p$ is a C-hyperoperation associated with $R_p$, for any $0 < p \leq 1$.

3. Basic Properties of F-C-Hyperoperations

In this section, we list some basic properties of F-C-hyperoperations.

Proposition 3.1. Let $(H, \ast_R)$ be a partial or nonpartial F-C-hypergroupoid defined on $H \neq \emptyset$. Then, for all $x, y, a, b \in H$, we have

$$x \ast_R y \cap a \ast_R b = x \ast_R b \cap a \ast_R y.$$  \hspace{1cm} (3.1)

Proof. For any $x, y, a, b, z \in H$, we have that $(x \ast_R y \cap a \ast_R b)(z) = (x \ast_R y)(z) \wedge (a \ast_R b)(z) = R(x, z) \wedge R(z, y) \wedge R(a, z) \wedge R(z, b) = R(x, z) \wedge R(z, b) \wedge R(a, z) \wedge R(z, y) = (x \ast_R b \cap a \ast_R y)(z)$.

So

$$x \ast_R y \cap a \ast_R b = x \ast_R b \cap a \ast_R y,$$  \hspace{1cm} (3.2)

for all $x, y, a, b \in H$. \hfill \Box
Proposition 3.2. Let $\langle H, \ast_R \rangle$ be a partial F-C-hypergroupoid and $x, y \in H, x \ast_R y = \emptyset$. Then,

1. $x \ast_R H \cap H \ast_R y = \emptyset$;
2. If $H = x \ast_R H$ then $H \ast_R y = \emptyset$;
3. If $H = H \ast_R x$ then $y \ast_R H = \emptyset$.

Proof. (1) Supposing $x \ast_R H \cap H \ast_R y \neq \emptyset$, then there exist $a, b \in H$, such that $x \ast_R a \cap b \ast_R y \neq \emptyset$. So from the previous proposition, we have $x \ast_R y \cap b \ast_R a \neq \emptyset$. This is a contradiction.

(2) From $H = x \ast_R H$ and $x \ast_R H \cap H \ast_R y = \emptyset$, we have that $H \cap H \ast_R y = \emptyset$, and so, $H \ast_R y = \emptyset$. (3) is proved similar to (2).

Proposition 3.3. Letting $\ast_R$ be the F-C-hyperoperation defined on the non-empty set $H$, $p \in (0, 1]$, then the following are equivalent:

1. for some $a \in H$, $(a \ast_R a)_p = H$;
2. for all $x, y \in H$, $a \in (x \ast_R y)_p$.

Proof. Let $(a \ast_R a)_p = H$. Then, for all $x, y \in H$, we have that $(a \ast_R a)(x) \geq p, (a \ast_R a)(y) \geq p$, that is $R(a, x) \geq p, R(x, a) \geq p, R(y, a) \geq p, R(a, y) \geq p$ and so $R(x, a) \land R(a, y) \geq p$. Thus $a \in (x \ast_R y)_p$ for all $x, y \in H$.

Conversely, let $a \in (x \ast_R y)_p$ for all $x, y \in H$. Specially, we have $a \in (a \ast_R x)_p$ and $a \in (x \ast_R a)_p$. Thus, $R(a, x) \geq p$ and $R(x, a) \geq p$. And so $x \in (a \ast_R a)_p$. □

Proposition 3.4. Let $\langle H, \ast_R \rangle$ be a partial or nonpartial F-C-hypergroupoid defined on $H \neq \emptyset$. Then, for all $a, b \in H, p \in (0, 1]$, we have

$$a \in (b \ast_R b)_p \iff b \in (a \ast_R a)_p. \quad (3.3)$$

Proof. For any $a, b \in H$, we have that

$$a \in (b \ast_R b)_p \implies (b \ast_R b)(a) \geq p \implies R(b, a) \land R(a, b) \geq p$$

$$(a \ast_R a)(b) \geq p \implies R(a, b) \land R(b, a) \geq p \implies (a \ast_R a)(b) \geq p \implies b \in (a \ast_R a)_p. \quad (3.4)$$

The remaining part can be proved similarly. □

4. F-C-Hyperoperations Associated with p-Fuzzy Reflexive Relations

In this section, we will assume that $R$ is a p-fuzzy reflexive relation on a non-empty set.

Definition 4.1. A fuzzy relation $R$ on a non-empty set $H$ is called p-fuzzy reflexive if for any $x \in H$,

$$R(x, x) \geq p. \quad (4.1)$$

Example 4.2. The fuzzy relation $R$ introduced in Example 2.3 is 0.1-fuzzy reflexive. Of course, it is p-fuzzy reflexive, where $0 \leq p \leq 0.1$. 
Corollary 4.6. Letting $(H, \ast_R)$ be a partial F-C-hypergroupoid defined on $H \neq \emptyset$, $R$ is $p$-fuzzy reflexive. Then, for all $a, b \in H$, $p \in (0, 1]$, the following are equivalent:

1. $R(a, b) \geq p$;
2. $a \in (a \ast_R b)_p$;
3. $b \in (a \ast_R b)_p$.

Proof. “(1)⇒(2)”
From $R(a, a) \geq p$ and $R(a, b) \geq p$ we have that $R(a, a) \land R(a, b) \geq p$ which shows that $a \in (a \ast_R b)_p$.

“(2)⇒(3)”
From $a \in (a \ast_R b)_p$ we have that $R(a, b) \geq p$. Since $R(b, b) \geq p$, so $R(a, b) \land R(b, b) \geq p$ which implies that $b \in (a \ast_R b)_p$.

“(3)⇒(1)”
It is obvious. □

Proposition 4.4. Letting $(H, \ast_R)$ be a partial F-C-hypergroupoid defined on $H \neq \emptyset$, $R$ is $p$-fuzzy reflexive. Then, for any $a \in H$, we have that

$$a \in (a \ast_R a)_p.$$  \hspace{1cm} (4.2)

Proof. From $R(a, a) \geq p$ we have $R(a, a) \land R(a, a) \geq p$. That is $a \in (a \ast_R a)_p$. □

Proposition 4.5. Letting $(H, \ast_R)$ be a partial F-C-hypergroupoid defined on $H \neq \emptyset$, $R$ is $p$-fuzzy reflexive. Then, for any $a, b \in H$, $p \in (0, 1]$, we have that

$$b \in (a \ast_R a)_p \iff a \in (a \ast_R b \land b \ast_R a)_p.$$  \hspace{1cm} (4.3)

Proof. From $b \in (a \ast_R a)_p$ we have that $R(a, b) \land R(b, a) \geq p$. So $R(a, b) \geq p$ and $R(b, a) \geq p$. Thus $R(a, a) \land R(a, b) \geq p$ and $R(b, a) \land R(a, a) \geq p$. That is $(a \ast_R b)(a) \geq p$ and $(b \ast_R a)(a) \geq p$. So $(a \ast_R b \land b \ast_R a)(a) \geq p$. Thus $a \in (a \ast_R b \land b \ast_R a)_p$.

Conversely, suppose that $a \in (a \ast_R b \land b \ast_R a)_p$. Then $(a \ast_R b)(a) \land (b \ast_R a)(a) \geq p$. Thus $R(a, a) \land R(a, b) \land R(b, a) \land R(a, a) \geq p$. So $R(a, b) \land R(b, a) \geq p$. That is $b \in (a \ast_R a)_p$. □

Corollary 4.6. Letting $(H, \ast_R)$ be a partial F-C-hypergroupoid defined on $H \neq \emptyset$, $R$ is $p$-fuzzy reflexive. Then, for any $a, b \in H$, $p \in (0, 1]$, we have that

$$b \in (a \ast_R a)_p \iff a \in (b \ast_R b)_p \iff a \in (a \ast_R b \land b \ast_R a)_p \iff b \in (a \ast_R b \land b \ast_R a)_p.$$  \hspace{1cm} (4.4)

Proposition 4.7. Letting $(H, \ast_R)$ be a partial F-C-hypergroupoid defined on $H \neq \emptyset$, $R$ is $p$-fuzzy reflexive. Then, for any $a, b \in H$, we have that

$$c \in (a \ast_R b)_p \iff c \in (a \ast_R c \land c \ast_R b)_p.$$  \hspace{1cm} (4.5)

Proof. If $c \in (a \ast_R b)_p$, then $R(a, c) \geq p$ and $R(c, b) \geq p$. Thus $c \in (a \ast_R c)_p$ and $c \in (c \ast_R b)_p$. So $c \in (a \ast_R c \land c \ast_R b)_p$. □
Conversely, if \( c \in (a \ast_R c \cap c \ast_R b)_p \), then \((a \ast_R c)(c) \wedge (c \ast_R b)(c) \geq p\). Thus \(R(a, c) \wedge R(c, c) \cap R(c, c) \wedge R(c, b) \geq p\). And so \(R(a, c) \wedge R(c, b) \geq p\). Thus \(c \in (a \ast_R b)_p\).

**Proposition 4.8.** Letting \((H, \ast_R)\) be a partial F-C-hypergroupoid defined on \(H \neq \emptyset\), \(R\) is \(p\)-fuzzy reflexive. Then, for any \(a, b, c \in H\), \(p \in (0, 1]\), the following are equivalent:

1. \(c \in (a \ast_R b)_p\);
2. \(a \in (a \ast_R c)_p\) and \(b \in (c \ast_R b)_p\);
3. \(a \in (a \ast_R c)_p\) and \(c \in (c \ast_R b)_p\).

**Proof.** “(1) \(\Rightarrow\) (2)”

Suppose that \(c \in (a \ast_R b)_p\). Then \(R(a, c) \geq p\) and \(R(c, b) \geq p\). So \(R(a, a) \wedge R(a, c) \geq p\) and \(R(c, b) \wedge R(b, b) \geq p\). Thus \(a \in (a \ast_R c)_p\) and \(b \in (c \ast_R b)_p\).

“(2) \(\Rightarrow\) (3)”

Suppose that \(b \in (c \ast_R b)_p\). Then \(R(c, b) \geq p\). Thus \(R(c, c) \wedge R(c, b) \geq p\). And so \(c \in (c \ast_R b)_p\).

“(3) \(\Rightarrow\) (1)”

From \(a \in (a \ast_R c)_p\) and \(c \in (c \ast_R b)_p\), we have that \(R(a, c) \geq p\) and \(R(c, b) \geq p\). Thus \(R(a, c) \wedge R(c, b) \geq p\). So \(c \in (a \ast_R b)_p\).

5. **F-C-Hyperoperations Associated with p-Fuzzy Symmetric Relations**

In this section, we will assume that \(R\) is a \(p\)-fuzzy symmetric relation on a non-empty set.

**Definition 5.1.** A fuzzy binary relation \(R\) on a non-empty set \(H\) is called \(p\)-fuzzy symmetric if for any \(x, y \in H\),

\[
R(x, y) \geq p \iff R(y, x) \geq p.
\] (5.1)

**Example 5.2.** The fuzzy relation \(R\) introduced in Example 2.3 is 0.2-fuzzy symmetric. Of course, it is \(p\)-fuzzy reflexive, where \(0 \leq p \leq 0.2\).

**Proposition 5.3.** Letting \((H, \ast_R)\) be a partial F-C-hypergroupoid defined on \(H \neq \emptyset\), \(R\) is \(p\)-fuzzy symmetric relation. Then, for all \(a, b \in H\), we have that

\[
(a \ast_R b)_p = (b \ast_R a)_p.
\] (5.2)

**Proof.** For all \(a, b \in H\), two cases are possible.

1. If \((a \ast_R b)_p = \emptyset\), then \((a \ast_R b)_p \subseteq (b \ast_R a)_p\).
2. If \((a \ast_R b)_p \neq \emptyset\), let \(x \in (a \ast_R b)_p\). Then \(R(a, x) \geq p\) and \(R(x, b) \geq p\).

Since \(R\) is \(p\)-fuzzy symmetric, so \(R(x, a) \geq p\) and \(R(b, x) \geq p\). Thus \((b \ast_R a)(x) = R(b, x) \wedge R(x, a) \geq p\). So \(x \in (b \ast_R a)_p\). And in this case, we also have that \((a \ast_R b)_p \subseteq (b \ast_R a)_p\).

The remaining part can be proved by exchanging \(a\) and \(b\).
Proposition 5.4. Let \( (H, \ast_R) \) be a partial F-C-hypergroupoid defined on \( H \neq \emptyset \), \( p \in (0, 1] \). if

1. for all \( a, b \in H \), \( (a \ast_R b)_p = (b \ast_R a)_p \).
2. for any \( x \in H \), there exists a \( y \in H \), such that \( R(x, y) \geq p \).

Then \( R \) is a \( p \)-fuzzy symmetric binary relation on \( H \).

Proof. For all \( a, b \in H \), suppose that \( R(a, b) \geq p \). We need to show that \( R(b, a) \geq p \).

Since for \( b \in H \), there exists an \( x \in H \), such that \( R(b, x) \geq p \). So \( R(a, b) \wedge R(b, x) \geq p \). That is, \( b \in (a \ast_R x)_p \). And so \( R(x, b) \wedge R(b, a) \geq p \). And finally we have that \( R(b, a) \geq p \).

\[ \square \]

6. F-C-Hyperoperations Associated with p-Fuzzy Transitive Relations

In this section, we will assume that \( R \) is a \( p \)-fuzzy transitive relation on a non-empty set.

Definition 6.1. A fuzzy binary relation \( R \) on a non-empty set \( H \) is called \( p \)-fuzzy transitive if for any \( x, y, z \in H \),

\[ R(x, y) \geq p, R(y, z) \geq p \implies R(x, z) \geq p. \]  

(6.1)

Example 6.2. The fuzzy relation \( R \) introduced in Example 2.3 is \( 0.1 \)-fuzzy transitive. Of course, it is \( p \)-fuzzy transitive, where \( 0 \leq p \leq 0.1 \).

Proposition 6.3. Letting \( (H, \ast_R) \) be a partial F-C-hypergroupoid defined on \( H \neq \emptyset \), \( R \) is a \( p \)-fuzzy transitive relation on \( H \), \( p \in (0, 1] \). Then for all \( x, y \in H \), we have that

\[ R(x, y) \geq p \implies (x \ast_R x \cup y \ast_R y)_p \subseteq (x \ast_R y)_p. \]  

(6.2)

Proof. (1) If \( (x \ast_R x)_p = \emptyset \), then obviously \( (x \ast_R x)_p \subseteq (x \ast_R y)_p \).

Supposing that \( (x \ast_R x)_p \neq \emptyset \), then for any \( w \in (x \ast_R x)_p \), we have that \( R(x, w) \wedge R(w, x) \geq p \), that is, \( R(x, w) \geq p \) and \( R(w, x) \geq p \). From \( R(w, x) \geq p \) and \( R(x, y) \geq p \) we have that \( R(w, y) \geq p \). From \( R(x, w) \geq p \) and \( R(w, y) \geq p \) we conclude that \( w \in (x \ast_R y)_p \).

So \( (x \ast_R x)_p \subseteq (x \ast_R y)_p \).

(2) If \( (y \ast_R y)_p = \emptyset \), then obviously \( (y \ast_R y)_p \subseteq (x \ast_R y)_p \).

Supposing that \( (y \ast_R y)_p \neq \emptyset \), then for any \( w \in (y \ast_R y)_p \), we have that \( R(y, w) \wedge R(w, y) \geq p \), that is, \( R(y, w) \geq p \) and \( R(w, y) \geq p \). From \( R(y, w) \geq p \) and \( R(x, y) \geq p \) we have that \( R(x, w) \geq p \). From \( R(x, w) \geq p \) and \( R(w, y) \geq p \) we conclude that \( w \in (x \ast_R y)_p \).

So \( (y \ast_R y)_p \subseteq (x \ast_R y)_p \).

\[ \square \]

Proposition 6.4. Letting \( (H, \ast_R) \) be a partial F-C-hypergroupoid defined on \( H \neq \emptyset \), \( R \) is a \( p \)-fuzzy transitive binary relation. For any \( a, b, c \in H \), we have that

1. \((a \ast_R b)_p \ast_R c)_p \subseteq (a \ast_R c)_p \),
2. \((a \ast_R b)_p \ast_R c)_p \subseteq (a \ast_R c)_p \).

(6.3)
Proof. (1) If \((a \ast_R b)_p \ast_R c_p = \emptyset\), then it is obvious that \(((a \ast_R b)_p \ast_R c_p) \subseteq (a \ast_R c)_p\).

Suppose that \(((a \ast_R b)_p \ast_R c_p) \neq \emptyset\). Then for any \(w \in ((a \ast_R b)_p \ast_R c_p)_p\), there exists a \(w_1 \in (a \ast_R b)_p\) such that \(w \in (w_1 \ast_R c)_p\). That is \(R(a, w_1) \geq p\), \(R(w_1, b) \geq p\), \(R(w_1, w) \geq p\) and \(R(w, c) \geq p\). From \(R(a, w_1) \geq p\) and \(R(w_1, w) \geq p\), we have that \(R(a, w) \geq p\). Thus \(R(a, w) \wedge R(w, c) \geq p \wedge p = p\). That is, \(w \in (a \ast_R c)_p\). So \(((a \ast_R b)_p \ast_R c_p)_p \subseteq (a \ast_R c)_p\).

(2) Can be proved similarly.

\(\square\)

Acknowledgment

The paper is partially supported by CSC.

References


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