SOME GENERATING FUNCTIONS OF MODIFIED BESSEL POLYNOMIALS FROM THE VIEW POINT OF LIE GROUP

ASIT KUMAR CHONGDAR

Department of Mathematics
Bangabasi Evening College
19 Scott Lane, Calcutta - 700 009.
India

(Received May 31, 1984)

ABSTRACT. In this paper we have derived a class of bilateral generating relation for modified Bessel polynomials from the view point of Lie group. An application of our theorem is also pointed out.

KEY WORDS AND PHRASES. Bessel Polynomials, Generating functions.

1980 MATHEMATICS SUBJECT CLASSIFICATION CODE. 33A70

1. INTRODUCTION.

In [1], the modified Bessel polynomials are defined by

\[ y_n(x; \alpha-n, \beta) = \sum_{n=0}^{\infty} a_n \frac{x^n}{n!} \]  \hspace{1cm} (1.1)

where \( y_n(x; \alpha, \beta) \) denotes the Bessel polynomials introduced by H. L. Krall and O. Frink [2].

The object of the present paper is to derive the following theorem on bilateral generating relation for modified Bessel polynomials from the view point of Lie-group.

THEOREM. If there exists a generating relation of the form,

\[ G(x, w) = \sum_{n=0}^{\infty} a_n w^n y_n(x; \alpha-n, \beta) \]  \hspace{1cm} (1.2)

then

\[ (1-wx)^{1-\alpha} e^{\beta w} G \left( \frac{x}{1-wx}, \alpha \right) = \sum_{n=0}^{\infty} w^n g_n(z) y_n(x; \alpha-n, \beta) \]  \hspace{1cm} (1.3)

where

\[ g_n(z) = \sum_{m=0}^{n} \frac{a_m}{m!} (\beta z)^m. \]

The importance of our result lies in the fact that if one knows a generating relation of the type (1.2) for a particular value of \( a_n \), then the corresponding bilateral generating relation follows at once from (1.3).

2. PROOF OF THE THEOREM.

From [1] we observe

\[ \exp(wR) f(x, y) = (1-wxy)^{1-\alpha} e^{\beta wy} f \left( \frac{x}{1-wxy}, y \right) \]  \hspace{1cm} (2.1)
where
\[ R = x^2 y \frac{\partial^2}{\partial x^2} + \beta y + (\alpha-1)xy \]
and
\[ R(F_n(x, y, \alpha-n, \beta)) = \beta F_{n+1}(x, y, \alpha-n-1, \beta) \tag{2.2} \]
where
\[ F_n(x, y, \alpha-n, \beta) = y^n y_n(x; \alpha-n, \beta). \]
In the formula:
\[ G(x, w) = \sum_{n=0}^{\infty} \frac{w^n}{n!} y_n(x; \alpha-n, \beta) \]
replacing \( wby wyz \) and then operating both sides by \( \exp(w\delta) \), we get
\[ \exp(w\delta) G(x, wyz) = \exp(w\delta) \sum_{n=0}^{\infty} \frac{a_n (wz)^n}{n!} F_n(x, y, \alpha-n, \beta). \]
The first member of (2.3) is equal to
\[ (1-wxy)^{1-\alpha} e^{\beta w} G(\frac{x}{1-wxy}, wyz), \]
and the second member of (2.3) is equal to
\[ \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{w^{n+m}}{n! m!} z^n \beta^m F_n(x, y, \alpha-n-m, \beta) \]
\[ = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{w^{n+m}}{n! m!} z^n \beta^m F_{n+m}(x, y, \alpha-n-m, \beta) \]
\[ = \sum_{n=0}^{\infty} (wy)^n g_n(z) y_n(x; \alpha-n, \beta) \]
where
\[ g_n(z) = \sum_{m=0}^{\infty} \frac{(\beta z)^m}{m!}. \]
Equating the above two results and then putting \( y=1 \), we get
\[ (1-wx)^{1-\alpha} e^{\beta w} G(\frac{x}{1-wx}, wz) = \sum_{n=0}^{\infty} \frac{w^n}{n!} g_n(z) y_n(x; \alpha-n, \beta) \]
where
\[ g_n(x) = \sum_{m=0}^{n} \frac{(\beta z)^m}{m!}, \]
this completes the proof of our theorem.

3. APPLICATION.

As an application of our theorem we consider the following generating relation [3].
\[ \sum_{n=0}^{\infty} \frac{w^n}{n!} y_n(x; \alpha-n, \beta) = e^{w (1- \frac{w}{\beta}) x} \] \[ l-\alpha \] \[ \tag{3.1} \]
If, in our theorem, we put \( a_n = 1/n! \) we obtain
\[ e^{(\beta+z)w (1- \frac{(\beta+z)w}{\beta}) x} = \sum_{n=0}^{\infty} \frac{w^n}{n!} g_n(z) y_n(x; \alpha-n, \beta) \] \[ l-\alpha \] \[ \tag{3.2} \]
where
\[ g_n(z) = \sum_{m=0}^{\infty} \frac{(\beta z)^m}{m!}. \]
REFERENCES

