ON FUZZY DOT SUBALGEBRAS OF BCH-ALGEBRAS

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(Received 21 December 2000)

ABSTRACT. We introduce the notion of fuzzy dot subalgebras in BCH-algebras, and study its various properties.

2000 Mathematics Subject Classification. 06F35, 03G25, 03E72.

1. Introduction. In [4], Hu and Li introduced the notion of BCH-algebras which are a generalization of BCK/BCI-algebras. In 1965, Zadeh [6] introduced the concept of fuzzy subsets. Since then several researchers have applied this notion to various mathematical disciplines. Jun [5] applied it to BCH-algebras, and he considered the fuzzification of ideals and filters in BCH-algebras. In this paper, we introduce the notion of a fuzzy dot subalgebra of a BCH-algebra as a generalization of a fuzzy subalgebra of a BCH-algebra, and then we investigate several basic properties related to fuzzy dot subalgebras.

2. Preliminaries. A BCH-algebra is an algebra \((X, \ast, 0)\) of type \((2,0)\) satisfying the following conditions:

(i) \(x \ast x = 0\),

(ii) \(x \ast y = 0 = y \ast x\) implies \(x = y\),

(iii) \((x \ast y) \ast z = (x \ast z) \ast y\) for all \(x, y, z \in X\).

In any BCH-algebra \(X\), the following hold (see [2]):

(P1) \(x \ast 0 = x\),

(P2) \(x \ast 0 = 0\) implies \(x = 0\),

(P3) \(0 \ast (x \ast y) = (0 \ast x) \ast (0 \ast y)\).

A BCH-algebra \(X\) is said to be medial if \(x \ast (x \ast y) = y\) for all \(x, y \in X\). A nonempty subset \(S\) of a BCH-algebra \(X\) is called a subalgebra of \(X\) if \(x \ast y \in S\) whenever \(x, y \in S\). A map \(f\) from a BCH-algebra \(X\) to a BCH-algebra \(Y\) is called a homomorphism if \(f(x \ast y) = f(x) \ast f(y)\) for all \(x, y \in X\).

We now review some fuzzy logic concepts. A fuzzy subset of a set \(X\) is a function \(\mu : X \rightarrow [0,1]\). For any fuzzy subsets \(\mu\) and \(\nu\) of a set \(X\), we define

\[
\mu \leq \nu \iff \mu(x) \leq \nu(x) \quad \forall x \in X,
\]

\[
(\mu \cap \nu)(x) = \min \{\mu(x), \nu(x)\} \quad \forall x \in X.
\]

Let \(f : X \rightarrow Y\) be a function from a set \(X\) to a set \(Y\) and let \(\mu\) be a fuzzy subset of \(X\).
The fuzzy subset $\nu$ of $Y$ defined by

$$
\nu(y) := \begin{cases} 
\sup_{x \in f^{-1}(y)} \mu(x) & \text{if } f^{-1}(y) \neq \emptyset, \forall y \in Y, \\
0 & \text{otherwise},
\end{cases}
$$

is called the image of $\mu$ under $f$, denoted by $f[\mu]$. If $\nu$ is a fuzzy subset of $Y$, the fuzzy subset $\mu$ of $X$ given by $\mu(x) = \nu(f(x))$ for all $x \in X$ is called the preimage of $\nu$ under $f$ and is denoted by $f^{-1}[\nu]$.

A fuzzy relation $\mu$ on a set $X$ is a fuzzy subset of $X \times X$, that is, a map $\mu : X \times X \to [0, 1]$. A fuzzy subset $\mu$ of a BCH-algebra $X$ is called a fuzzy subalgebra of $X$ if

$$
\mu(x \ast y) \geq \min\{\mu(x), \mu(y)\}
$$

for all $x, y \in X$.

3. Fuzzy product subalgebras. In what follows let $X$ denote a BCH-algebra unless otherwise specified.

**Definition 3.1.** A fuzzy subset $\mu$ of $X$ is called a fuzzy dot subalgebra of $X$ if

$$
\mu(x \ast y) \geq \mu(x) \cdot \mu(y)
$$

for all $x, y \in X$.

**Example 3.2.** Consider a BCH-algebra $X = \{0, a, b, c\}$ having the following Cayley table (see [1]):

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>a</td>
<td>a</td>
<td>0</td>
<td>0</td>
<td>a</td>
</tr>
<tr>
<td>b</td>
<td>b</td>
<td>c</td>
<td>0</td>
<td>c</td>
</tr>
<tr>
<td>c</td>
<td>c</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Define a fuzzy set $\mu$ in $X$ by $\mu(0) = 0.5$, $\mu(a) = 0.6$, $\mu(b) = 0.4$, $\mu(c) = 0.3$. It is easy to verify that $\mu$ is a fuzzy dot subalgebra of $X$.

Note that every fuzzy subalgebra is a fuzzy dot subalgebra, but the converse is not true. In fact, the fuzzy dot subalgebra $\mu$ in Example 3.2 is not a fuzzy subalgebra since

$$
\mu(a \ast a) = \mu(0) = 0.5 < 0.6 = \mu(a) = \min \{\mu(a), \mu(a)\}. 
$$

**Proposition 3.3.** If $\mu$ is a fuzzy dot subalgebra of $X$, then

$$
\mu(0) \geq (\mu(x))^2, \quad \mu(0^n \ast x) \geq (\mu(x))^{2n+1},
$$

for all $x \in X$ and $n \in \mathbb{N}$ where $0^n \ast x = 0 \ast (0 \ast (\cdots (0 \ast x) \cdots))$ in which $0$ occurs $n$ times.

**Proof.** Since $x \ast x = 0$ for all $x \in X$, it follows that

$$
\mu(0) = \mu(x \ast x) \geq \mu(x) \cdot \mu(x) = (\mu(x))^2
$$

for all $x \in X$. The proof of the second part is by induction on $n$. For $n = 1$, we have $\mu(0 \ast x) \geq \mu(0) \cdot \mu(x) \geq (\mu(x))^3$ for all $x \in X$. Assume that $\mu(0^k \ast x) \geq (\mu(x))^{2k+1}$ for
all \( x \in X \). Then

\[
\mu(0^{k+1} \ast x) = \mu(0 \ast (0^k \ast x)) \geq \mu(0) \cdot \mu(0^k \ast x) \\
\geq (\mu(x))^2 \cdot (\mu(x))^{2k+1} = (\mu(x))^{2(k+1)+1}.
\]

(3.4)

Hence \( \mu(0^n \ast x) \geq (\mu(x))^{2n+1} \) for all \( x \in X \) and \( n \in \mathbb{N} \).

**Proposition 3.4.** Let \( \mu \) be a fuzzy dot subalgebra of \( X \). If there exists a sequence \( \{x_n\} \) in \( X \) such that \( \lim_{n \to \infty} (\mu(x_n))^2 = 1 \), then \( \mu(0) = 1 \).

**Proof.** According to Proposition 3.3, \( \mu(0) \geq (\mu(x_n))^2 \) for each \( n \in \mathbb{N} \). Since \( 1 \geq \mu(0) \geq \lim_{n \to \infty} (\mu(x_n))^2 = 1 \), it follows that \( \mu(0) = 1 \).

**Theorem 3.5.** If \( \mu \) and \( \nu \) are fuzzy dot subalgebras of \( X \), then so is \( \mu \cap \nu \).

**Proof.** Let \( x, y \in X \), then

\[
(\mu \cap \nu)(x \ast y) = \min \{\mu(x \ast y), \nu(x \ast y)\} \\
\geq \min \{\mu(x) \cdot \mu(y), \nu(x) \cdot \nu(y)\} \\
\geq (\min \{\mu(x), \nu(x)\}) \cdot (\min \{\mu(y), \nu(y)\}) \\
= ((\mu \cap \nu)(x)) \cdot ((\mu \cap \nu)(y)).
\]

(3.5)

Hence \( \mu \cap \nu \) is a fuzzy dot subalgebra of \( X \).

Note that a fuzzy subset \( \mu \) of \( X \) is a fuzzy subalgebra of \( X \) if and only if a nonempty level subset

\[
U(\mu; t) := \{x \in X \mid \mu(x) \geq t\}
\]

(3.6)

is a subalgebra of \( X \) for every \( t \in [0, 1] \). But, we know that if \( \mu \) is a fuzzy dot subalgebra of \( X \), then there exists \( t \in [0, 1] \) such that

\[
U(\mu; t) := \{x \in X \mid \mu(x) \geq t\}
\]

(3.7)

is not a subalgebra of \( X \). In fact, if \( \mu \) is the fuzzy dot subalgebra of \( X \) in Example 3.2, then

\[
U(\mu; 0.4) = \{x \in X \mid \mu(x) \geq 0.4\} = \{0, a, b\}
\]

(3.8)

is not a subalgebra of \( X \) since \( b \ast a = c \notin U(\mu; 0.4) \).

**Theorem 3.6.** If \( \mu \) is a fuzzy dot subalgebra of \( X \), then

\[
U(\mu; 1) := \{x \in X \mid \mu(x) = 1\}
\]

(3.9)

is either empty or is a subalgebra of \( X \).

**Proof.** If \( x \) and \( y \) belong to \( U(\mu; 1) \), then \( \mu(x \ast y) \geq \mu(x) \cdot \mu(y) = 1 \). Hence \( \mu(x \ast y) = 1 \) which implies \( x \ast y \in U(\mu; 1) \). Consequently, \( U(\mu; 1) \) is a subalgebra of \( X \).
**Theorem 3.7.** Let $X$ be a medial BCH-algebra and let $\mu$ be a fuzzy subset of $X$ such that
\[
\mu(0 \ast x) \geq \mu(x), \quad \mu(x \ast (0 \ast y)) \geq \mu(x) \cdot \mu(y),
\]
for all $x, y \in X$. Then $\mu$ is a fuzzy dot subalgebra of $X$.

**Proof.** Since $X$ is medial, we have $0 \ast (0 \ast y) = y$ for all $y \in X$. Hence
\[
\mu(x \ast y) = \mu(x \ast (0 \ast (0 \ast y))) \geq \mu(x) \cdot \mu(0 \ast y) \geq \mu(x) \cdot \mu(y)
\]
for all $x, y \in X$. Therefore $\mu$ is a fuzzy dot subalgebra of $X$.

**Theorem 3.8.** Let $g : X \to Y$ be a homomorphism of BCH-algebras. If $\nu$ is a fuzzy dot subalgebra of $Y$, then the preimage $g^{-1}[\nu]$ of $\nu$ under $g$ is a fuzzy dot subalgebra of $X$.

**Proof.** For any $x_1, x_2 \in X$, we have
\[
g^{-1}[\nu](x_1 \ast x_2) = \nu(g(x_1 \ast x_2)) = \nu(g(x_1) \ast g(x_2))
\]
\[
\geq \nu(g(x_1)) \cdot \nu(g(x_2)) = g^{-1}[\nu](x_1) \cdot g^{-1}[\nu](x_2).
\]
Thus $g^{-1}[\nu]$ is a fuzzy dot subalgebra of $X$.

**Theorem 3.9.** Let $f : X \to Y$ be an onto homomorphism of BCH-algebras. If $\mu$ is a fuzzy dot subalgebra of $X$, then the image $f[\mu]$ of $\mu$ under $f$ is a fuzzy dot subalgebra of $Y$.

**Proof.** For any $y_1, y_2 \in Y$, let $A_1 = f^{-1}(y_1)$, $A_2 = f^{-1}(y_2)$, and $A_{12} = f^{-1}(y_1 \ast y_2)$. Consider the set
\[
A_1 \ast A_2 := \{x \in X \mid x = a_1 \ast a_2 \text{ for some } a_1 \in A_1, a_2 \in A_2\}.
\]
If $x \in A_1 \ast A_2$, then $x = x_1 \ast x_2$ for some $x_1 \in A_1$ and $x_2 \in A_2$ so that
\[
f(x) = f(x_1 \ast x_2) = f(x_1) \ast f(x_2) = y_1 \ast y_2,
\]
that is, $x \in f^{-1}(y_1 \ast y_2) = A_{12}$. Hence $A_1 \ast A_2 \subseteq A_{12}$. It follows that
\[
f[\mu](y_1 \ast y_2) = \sup_{x \in f^{-1}(y_1 \ast y_2)} \mu(x) = \sup_{x \in A_{12}} \mu(x)
\]
\[
\geq \sup_{x \in A_1 \ast A_2} \mu(x) \geq \sup_{x_1 \in A_1, x_2 \in A_2} \mu(x_1 \ast x_2)
\]
\[
\geq \sup_{x_1 \in A_1, x_2 \in A_2} \mu(x_1) \cdot \mu(x_2).
\]
Since $\cdot : [0,1] \times [0,1] \to [0,1]$ is continuous, for every $\varepsilon > 0$ there exists $\delta > 0$ such that if $\tilde{x}_1 \geq \sup_{x_1 \in A_1} \mu(x_1) - \delta$ and $\tilde{x}_2 \geq \sup_{x_2 \in A_2} \mu(x_2) - \delta$, then $\tilde{x}_1 \cdot \tilde{x}_2 \geq \sup_{x_1 \in A_1} \mu(x_1) \cdot \sup_{x_2 \in A_2} \mu(x_2) - \varepsilon$. Choose $a_1 \in A_1$ and $a_2 \in A_2$ such that $\mu(a_1) \geq \cdots$
\[ \sup_{x_1 \in A_1} \mu(x_1) - \delta \text{ and } \mu(a_2) \leq \sup_{x_2 \in A_2} \mu(x_2) - \delta. \] Then
\[ \mu(a_1) \cdot \mu(a_2) \geq \sup_{x_1 \in A_1} \mu(x_1) \cdot \sup_{x_2 \in A_2} \mu(x_2) - \varepsilon. \quad (3.16) \]

Consequently,
\[ f[\mu](y_1 \cdot y_2) \geq \sup_{x_1 \in A_1, x_2 \in A_2} \mu(x_1) \cdot \mu(x_2) \]
\[ \geq \sup_{x_1 \in A_1} \mu(x_1) \cdot \sup_{x_2 \in A_2} \mu(x_2) \quad (3.17) \]
\[ = f[\mu](y_1) \cdot f[\mu](y_2), \]

and hence \( f[\mu] \) is a fuzzy dot subalgebra of \( Y \).

**Definition 3.10.** Let \( \sigma \) be a fuzzy subset of \( X \). The strongest fuzzy \( \sigma \)-relation on \( X \) is the fuzzy subset \( \mu_\sigma \) of \( X \times X \) given by \( \mu_\sigma(x, y) = \sigma(x) \cdot \sigma(y) \) for all \( x, y \in X \).

**Theorem 3.11.** Let \( \mu_\sigma \) be the strongest fuzzy \( \sigma \)-relation on \( X \), where \( \sigma \) is a fuzzy subset of \( X \). If \( \sigma \) is a fuzzy dot subalgebra of \( X \), then \( \mu_\sigma \) is a fuzzy dot subalgebra of \( X \times X \).

**Proof.** Assume that \( \sigma \) is a fuzzy dot subalgebra of \( X \). For any \( x_1, x_2, y_1, y_2 \in X \), we have
\[ \mu_\sigma((x_1, y_1) \cdot (x_2, y_2)) = \mu_\sigma(x_1 \cdot x_2, y_1 \cdot y_2) \]
\[ = \sigma(x_1 \cdot x_2) \cdot \sigma(y_1 \cdot y_2) \]
\[ \geq (\sigma(x_1) \cdot \sigma(x_2)) \cdot (\sigma(y_1) \cdot \sigma(y_2)) \quad (3.18) \]
\[ = (\sigma(x_1) \cdot \sigma(y_1)) \cdot (\sigma(x_2) \cdot \sigma(y_2)) \]
\[ = \mu_\sigma(x_1, y_1) \cdot \mu_\sigma(x_2, y_2), \]

and so \( \mu_\sigma \) is a fuzzy dot subalgebra of \( X \times X \).

**Definition 3.12.** Let \( \sigma \) be a fuzzy subset of \( X \). A fuzzy relation \( \mu \) on \( X \) is called a fuzzy \( \sigma \)-product relation if \( \mu(x, y) \geq \sigma(x) \cdot \sigma(y) \) for all \( x, y \in X \).

**Definition 3.13.** Let \( \sigma \) be a fuzzy subset of \( X \). A fuzzy relation \( \mu \) on \( X \) is called a left fuzzy relation on \( \sigma \) if \( \mu(x, y) = \sigma(x) \) for all \( x, y \in X \).

Similarly, we can define a right fuzzy relation on \( \sigma \). Note that a left (resp., right) fuzzy relation on \( \sigma \) is a fuzzy \( \sigma \)-product relation.

**Theorem 3.14.** Let \( \mu \) be a left fuzzy relation on a fuzzy subset \( \sigma \) of \( X \). If \( \mu \) is a fuzzy dot subalgebra of \( X \times X \), then \( \sigma \) is a fuzzy dot subalgebra of \( X \).

**Proof.** Assume that a left fuzzy relation \( \mu \) on \( \sigma \) is a fuzzy dot subalgebra of \( X \times X \). Then
\[ \sigma(x_1 \cdot x_2) = \mu(x_1 \cdot x_2, y_1 \cdot y_2) = \mu((x_1, y_1) \cdot (x_2, y_2)) \]
\[ \geq \mu(x_1, y_1) \cdot \mu(x_2, y_2) = \sigma(x_1) \cdot \sigma(x_2) \quad (3.19) \]
for all \( x_1, x_2, y_1, y_2 \in X \). Hence \( \sigma \) is a fuzzy dot subalgebra of \( X \).
Theorem 3.15. Let $\mu$ be a fuzzy relation on $X$ satisfying the inequality $\mu(x,y) \leq \mu(x,0)$ for all $x,y \in X$. Given $z \in X$, let $\sigma_z$ be a fuzzy subset of $X$ defined by $\sigma_z(x) = \mu(x,z)$ for all $x \in X$. If $\mu$ is a fuzzy dot subalgebra of $X \times X$, then $\sigma_z$ is a fuzzy dot subalgebra of $X$ for all $z \in X$.

Proof. Let $z,x,y \in X$, then
\[ \sigma_z(x \ast y) = \mu(x \ast y, z) = \mu((x,z) \ast (y,0)) \geq \mu(x,z) \cdot \mu(y,0) \]
\[ \geq \mu(x,z) \cdot \mu(y,z) = \sigma_z(x) \cdot \sigma_z(y), \]
completing the proof.

Theorem 3.16. Let $\mu$ be a fuzzy relation on $X$ and let $\sigma_{\mu}$ be a fuzzy subset of $X$ given by $\sigma_{\mu}(x) = \inf_{y \in X} \mu(x,y) \cdot \mu(y,x)$ for all $x \in X$. If $\mu$ is a fuzzy dot subalgebra of $X \times X$ satisfying the equality $\mu(x,0) = 1 = \mu(0,x)$ for all $x \in X$, then $\sigma_{\mu}$ is a fuzzy dot subalgebra of $X$.

Proof. For any $x,y,z \in X$, we have
\[ \mu(x \ast y, z) = \mu(x \ast y, z \ast 0) = \mu((x,z) \ast (y,0)) \]
\[ \geq \mu(x,z) \cdot \mu(y,0) = \mu(x,z), \]
\[ \mu(z, x \ast y) = \mu(z \ast 0, x \ast y) = \mu((z,x) \ast (0,y)) \]
\[ \geq \mu(z,x) \cdot \mu(0,y) = \mu(z,x). \]
It follows that
\[ \mu(x \ast y, z) \cdot \mu(z, x \ast y) \geq \mu(x,z) \cdot \mu(z,x) \]
\[ \geq (\mu(x,z) \cdot \mu(z,x)) \cdot (\mu(y,z) \cdot \mu(z,y)) \]
so that
\[ \sigma_{\mu}(x \ast y) = \inf_{z \in X} \mu(x \ast y, z) \cdot \mu(z, x \ast y) \]
\[ \geq \left( \inf_{z \in X} \mu(x,z) \cdot \mu(z,x) \right) \cdot \left( \inf_{z \in X} \mu(y,z) \cdot \mu(z,y) \right) \]
\[ = \sigma_{\mu}(x) \cdot \sigma_{\mu}(y). \]
This completes the proof.

Definition 3.17 (see Choudhury et al. [3]). A fuzzy map $f$ from a set $X$ to a set $Y$ is an ordinary map from $X$ to the set of all fuzzy subsets of $Y$ satisfying the following conditions:

(C1) for all $x \in X$, there exists $y_x \in X$ such that $(f(x))(y_x) = 1$,
(C2) for all $x \in X$, $f(x)(y_1) = f(x)(y_2)$ implies $y_1 = y_2$.

One observes that a fuzzy map $f$ from $X$ to $Y$ gives rise to a unique ordinary map $\mu_f : X \times X \rightarrow I$, given by $\mu_f(x,y) = f(x)(y)$. One also notes that a fuzzy map from $X$ to $Y$ gives a unique ordinary map $f_1 : X \rightarrow Y$ defined as $f_1(x) = y_x$. 
**Definition 3.18.** A fuzzy map \( f \) from a BCH-algebra \( X \) to a BCH-algebra \( Y \) is called a fuzzy homomorphism if

\[
\mu_f(x_1 \ast x_2, y) = \sup_{y = y_1 \ast y_2} \mu_f(x_1, y_1) \cdot \mu_f(x_2, y_2)
\]

for all \( x_1, x_2 \in X \) and \( y \in Y \).

One notes that if \( f \) is an ordinary map, then the above definition reduces to an ordinary homomorphism. One also observes that if a fuzzy map \( f \) is a fuzzy homomorphism, then the induced ordinary map \( f_1 \) is an ordinary homomorphism.

**Proposition 3.19.** Let \( f : X \to Y \) be a fuzzy homomorphism of BCH-algebras. Then

(i) \( \mu_f(x_1 \ast x_2, y_1 \ast y_2) \geq \mu_f(x_1, y_1) \cdot \mu_f(x_2, y_2) \) for all \( x_1, x_2 \in X \) and \( y_1, y_2 \in Y \).

(ii) \( \mu_f(0, 0) = 1 \).

(iii) \( \mu_f(0 \ast x, 0 \ast y) \geq \mu_f(x, y) \) for all \( x \in X \) and \( y \in Y \).

(iv) If \( Y \) is medial and \( \mu_f(x, y) = t \neq 0 \), then \( \mu_f(0, y_x \ast y) = t \) for all \( x \in X \) and \( y \in Y \), where \( y_x \in Y \) with \( \mu_f(x, y_x) = 1 \).

**Proof.** (i) For every \( x_1, x_2 \in X \) and \( y_1, y_2 \in Y \), we have

\[
\mu_f(x_1 \ast x_2, y_1 \ast y_2) \leq \sup_{y_1 \ast y_2 = y_1 \ast y_2} \mu_f(x_1, y_1) \cdot \mu_f(x_2, y_2)
\]

and so \( \mu_f(0, 0) = 1 \).

(iii) The proof follows from (i) and (ii).

(iv) Assume that \( Y \) is medial and \( \mu_f(x, y) = t \neq 0 \) for all \( x \in X \) and \( y \in Y \), and let \( y_x \in Y \) be such that \( \mu_f(x, y_x) = 1 \). Then

\[
\mu_f(0, y_x \ast y) = \mu_f(x \ast 0, y_x \ast y) \geq \mu_f(x, y_x) \cdot \mu_f(x, y)
\]

\[
= t = \mu_f(x, y) = \mu_f(x \ast 0, y_x \ast (y_x \ast y))
\]

\[
\geq \mu_f(x, y_x) \cdot \mu_f(0, y_x \ast y) = \mu_f(0, y_x \ast y),
\]

and hence \( \mu_f(0, y_x \ast y) = t \). This completes the proof.

**Acknowledgement.** This work was supported by Korea Research Foundation Grant (KRF-99-005-D00003).

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