INTUITIONISTIC FUZZY INTERIOR IDEALS OF SEMIGROUPS

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ABSTRACT. We consider the intuitionistic fuzzification of the concept of interior ideals in a semigroup $S$, and investigate some properties of such ideals. For any homomorphism $f$ from a semigroup $S$ to a semigroup $T$, if $B = (\mu_B, \gamma_B)$ is an intuitionistic fuzzy interior ideal of $T$, then the preimage $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$ of $B$ under $f$ is an intuitionistic fuzzy interior ideal of $S$.

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1. Introduction. The idea of “intuitionistic fuzzy set” was first published by Atanassov [1, 2], as a generalization of the notion of fuzzy set. Jun et al. considered the fuzzification of interior ideals in semigroups [3]. In this paper, we introduce the notion of an intuitionistic fuzzy interior ideal of a semigroup $S$, and then some related properties are investigated. Characterizations of intuitionistic fuzzy interior ideals are given. Also for any homomorphism $f$ from a semigroup $S$ to a semigroup $T$, if $B = (\mu_B, \gamma_B)$ is an intuitionistic fuzzy interior ideal of $T$, then the preimage $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$ of $B$ under $f$ is an intuitionistic fuzzy interior ideal of $S$.

2. Preliminaries. Let $X$ be a nonempty fixed set. An intuitionistic fuzzy set (IFS for short) $A$ is an object having the form

$$A = \{(x, \mu_A(x), \gamma_A(x)) : x \in X\},$$

(2.1)

where the functions $\mu_A : X \to [0, 1]$ and $\gamma_A : X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) of each element $x \in X$ to the set $A$, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for all $x \in X$ (see Atanassov [1, 2]). For the sake of simplicity, we use the symbol $A = (\mu_A, \gamma_A)$ for the IFS $A = \{(x, \mu_A(x), \gamma_A(x)) : x \in X\}$.

Let $S$ be a semigroup. By a subsemigroup of $S$ we mean a nonempty subset $A$ of $S$ such that $A^2 \subseteq A$. A subsemigroup $A$ of a semigroup $S$ is called an interior ideal of $S$ if $SAS \subseteq A$. A mapping $f$ from a semigroup $S$ to a semigroup $T$ is called a homomorphism if $f(xy) = f(x)f(y)$ for all $x, y \in S$.

A fuzzy set $\mu$ in a semigroup $S$ is called a fuzzy subsemigroup of $S$ (see [3]) if $\mu(xy) \geq \mu(x) \wedge \mu(y)$ for all $x, y \in S$.

A fuzzy subsemigroup $\mu$ of a semigroup $S$ is called a fuzzy interior ideal of $S$ (see [3]) if $\mu(xay) \geq \mu(a)$ for all $a, x, y \in S$. 
3. Intuitionistic fuzzy interior ideals. In what follows, $S$ denotes a semigroup unless otherwise specified.

**Definition 3.1.** An IFS $A = (\mu_A, \gamma_A)$ in $S$ is called an intuitionistic fuzzy subsemigroup of $S$ if it satisfies

(IF1) $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$,

(IF2) $\gamma_A(xy) \leq \gamma_A(x) \vee \gamma_A(y)$,

for all $x, y \in S$.

**Example 3.2.** Let $S = \{0, e, f, a, b\}$ be a set with the following Cayley table:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>e</th>
<th>f</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
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<tr>
<td>e</td>
<td>0</td>
<td>e</td>
<td>0</td>
<td>a</td>
<td>0</td>
</tr>
<tr>
<td>f</td>
<td>0</td>
<td>0</td>
<td>f</td>
<td>0</td>
<td>b</td>
</tr>
<tr>
<td>a</td>
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<td>a</td>
<td>0</td>
<td>0</td>
<td>e</td>
</tr>
<tr>
<td>b</td>
<td>0</td>
<td>0</td>
<td>b</td>
<td>f</td>
<td>0</td>
</tr>
</tbody>
</table>

Then $S$ is a semigroup (see [4]). Define an IFS $A = (\mu_A, \gamma_A)$ in $S$ by $\mu_A(0) = \mu_A(e) = \mu_A(f) = 1$, $\mu_A(a) = \mu_A(b) = 0$, $\gamma_A(0) = \gamma_A(e) = \gamma_A(f) = 0$, and $\gamma_A(a) = \gamma_A(b) = 1$. By routine calculations we know that $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy subsemigroup of $S$.

**Definition 3.3.** An intuitionistic fuzzy subsemigroup $A = (\mu_A, \gamma_A)$ of $S$ is called an intuitionistic fuzzy interior ideal of $S$ if

(IF3) $\mu_A(xay) \geq \mu_A(a)$,

(IF4) $\gamma_A(xay) \leq \gamma_A(a)$,

for all $x, y, a \in S$.

**Example 3.4.** The IFS $A = (\mu_A, \gamma_A)$ in Example 3.2 is an intuitionistic fuzzy interior ideal of $S$.

**Theorem 3.5.** If $\{A_i\}_{i \in \Lambda}$ is a family of intuitionistic fuzzy interior ideals of $S$, then $\bigcap A_i$ is an intuitionistic fuzzy interior ideal of $S$, where $\bigcap A_i = (\bigwedge_{A_i} \mu_{A_i}, \bigvee_{A_i} \gamma_{A_i})$ and $\bigwedge_{A_i}$ and $\bigvee_{A_i}$ are defined as follows:

$$\bigwedge_{A_i}(x) = \inf \{\mu_{A_i}(x) \mid i \in \Lambda, x \in S\},$$

$$\bigvee_{A_i}(x) = \sup \{\gamma_{A_i}(x) \mid i \in \Lambda, x \in S\}.$$  

(3.1)

**Proof.** Let $x, y, a \in S$. Then

$$\bigwedge_{A_i}(xy) \geq \bigwedge_{A_i}(x) \wedge \bigwedge_{A_i}(y) = (\bigwedge_{A_i}(x)) \wedge (\bigwedge_{A_i}(y)),$$

$$\bigvee_{A_i}(xy) \leq \bigvee_{A_i}(x) \vee \bigvee_{A_i}(y) = (\bigvee_{A_i}(x)) \vee (\bigvee_{A_i}(y)),$$

$$\bigwedge_{A_i}(xay) \geq \bigwedge_{A_i}(a), \quad \bigvee_{A_i}(xay) \leq \bigvee_{A_i}(a).$$  

(3.2)

Hence $\bigcap A_i$ is an intuitionistic fuzzy interior ideal of $S$. 

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**Theorem 3.6.** If an IFS \( A = (\mu_A, y_A) \) in \( S \) is an intuitionistic fuzzy interior ideal of \( S \), then so is \( \square A := (\mu_A, \bar{\mu}_A), \bar{\mu}_A = 1 - \mu_A \).

**Proof.** It is sufficient to show that \( \bar{\mu}_A \) satisfies conditions (IF2) and (IF4). For any \( a, x, y \in S \), we have

\[
\bar{\mu}_A(xy) = 1 - \mu_A(xy) = 1 - (\mu_A(x) \land \mu_A(y))
\]

and \( \bar{\mu}_A(xa) = 1 - \mu_A(xa) \leq 1 - \mu_A(a) = \bar{\mu}_A(a) \). Therefore, \( A \) is an intuitionistic fuzzy interior ideal of \( S \).

**Definition 3.7.** Let \( A = (\mu_A, y_A) \) be an IFS in \( S \) and let \( \alpha \in [0,1] \). Then the sets

\[
\mu_{\hat{A},\alpha} := \{ x \in S : \mu_A(x) \geq \alpha \}, \quad y_{\hat{A},\alpha} := \{ x \in S : y_A(x) \leq \alpha \}
\]

are called a \( \mu \)-level \( \alpha \)-cut and a \( y \)-level \( \alpha \)-cut of \( A \), respectively.

**Theorem 3.8.** If an IFS \( A = (\mu_A, y_A) \) in \( S \) is an intuitionistic fuzzy interior ideal of \( S \), then the \( \mu \)-level \( \alpha \)-cut \( \mu_{\hat{A},\alpha} \) and \( y \)-level \( \alpha \)-cut \( y_{\hat{A},\alpha} \) of \( A \) are interior ideals of \( S \) for every \( \alpha \in \text{Im}(\mu_A) \cap \text{Im}(y_A) \subseteq [0,1] \).

**Proof.** Let \( \alpha \in \text{Im}(\mu_A) \cap \text{Im}(y_A) \subseteq [0,1] \) and let \( x, y \in \mu_{\hat{A},\alpha} \). Then \( \mu_A(x) \geq \alpha \) and \( \mu_A(y) \geq \alpha \). It follows from (IF1) that

\[
\mu_A(xy) \geq \mu_A(x) \land \mu_A(y) \geq \alpha \quad \text{so that} \quad xy \in \mu_{\hat{A},\alpha}.
\]

If \( x, y \in y_{\hat{A},\alpha} \), then \( y_A(x) \leq \alpha \) and \( y_A(y) \leq \alpha \), and so

\[
y_A(xy) \leq y_A(x) \lor y_A(y) \leq \alpha, \quad \text{that is} \quad xy \in y_{\hat{A},\alpha}.
\]

Hence \( \mu_{\hat{A},\alpha} \) and \( y_{\hat{A},\alpha} \) are subsemigroups of \( S \). Now let \( x, y \in S \) and \( a \in \mu_{\hat{A},\alpha} \). Then \( \mu_A(xa) \geq \mu_A(a) \geq \alpha \) and so \( xa \in \mu_{\hat{A},\alpha} \). If \( a \in y_{\hat{A},\alpha} \), then \( y_A(xa) \leq y_A(a) \leq \alpha \) and thus \( xa \in y_{\hat{A},\alpha} \). Therefore \( \mu_{\hat{A},\alpha} \) and \( y_{\hat{A},\alpha} \) are interior ideals of \( S \).

**Theorem 3.9.** Let \( A = (\mu_A, y_A) \) be an IFS in \( S \) such that the nonempty sets \( \mu_{\hat{A},\alpha} \) and \( y_{\hat{A},\alpha} \) are interior ideals of \( S \) for all \( \alpha \in [0,1] \). Then \( A = (\mu_A, y_A) \) is an intuitionistic fuzzy interior ideal of \( S \).

**Proof.** Let \( \alpha \in [0,1] \) and suppose that \( \mu_{\hat{A},\alpha}(\neq \emptyset) \) and \( y_{\hat{A},\alpha}(\neq \emptyset) \) are interior ideals of \( S \). We must show that \( A = (\mu_A, y_A) \) satisfies conditions (IF1)–(IF4). If condition (IF1) is false, then there exist \( x_0, y_0 \in S \) such that \( \mu_A(x_0 y_0) < \mu_A(x_0) \land \mu_A(y_0) \). Taking

\[
\alpha_0 := \frac{1}{2} (\mu_A(x_0 y_0) + \mu_A(x_0) \land \mu_A(y_0)),
\]

we have \( \mu_A(x_0 y_0) < \alpha_0 < \mu_A(x_0) \land \mu_A(y_0) \). It follows that \( x_0, y_0 \in \mu_{\hat{A},\alpha_0} \) and \( x_0 y_0 \notin \mu_{\hat{A},\alpha_0} \), which is a contradiction. Hence condition (IF1) is true. The proof of other conditions are similar to the case (IF1), we omit the proof.
Theorem 3.10. Let \( M \) be an interior ideal of \( S \) and let \( A = (\mu_A, \gamma_A) \) be an IFS in \( S \) defined by

\[
\mu_A(x) := \begin{cases} 
\alpha_0 & \text{if } x \in M, \\
\alpha_1 & \text{otherwise},
\end{cases}
\]

\[
\gamma_A(x) := \begin{cases} 
\beta_0 & \text{if } x \in M, \\
\beta_1 & \text{otherwise},
\end{cases}
\]

(3.8)

for all \( x \in S \) and \( \alpha_1, \beta_1 \in [0, 1] \) such that \( \alpha_0 > \alpha_1, \beta_0 < \beta_1, \) and \( \alpha_1 + \beta_1 = 1 \) for \( i = 0, 1 \). Then \( A = (\mu_A, \gamma_A) \) is an intuitionistic fuzzy interior ideal of \( S \) and \( \mu_A^\alpha \beta_0 = M = \gamma_A^\alpha \beta_0 \).

Proof. Let \( x, y \in S \). If anyone of \( x \) and \( y \) does not belong to \( M \), then

\[
\mu_A(xy) \geq \alpha_1 = \mu_A(x) \land \mu_A(y),
\]

\[
\gamma_A(xy) \leq \beta_1 = \gamma_A(x) \lor \gamma_A(y).
\]

(3.9)

Other cases are trivial, and we omit the proof. Hence \( A = (\mu_A, \gamma_A) \) is an intuitionistic fuzzy subsemigroup of \( S \). Now let \( x, y, a \in S \). If \( a \notin M \), then \( \mu_A(xay) \geq \alpha_1 = \mu_A(a) \) and \( \gamma_A(xay) \leq \beta_1 = \gamma_A(a) \). Assume that \( a \in M \). Since \( M \) is an interior ideal of \( S \), it follows that \( xay \in M \). Hence \( \mu_A(xay) = \alpha_0 = \mu_A(a) \) and \( \gamma_A(xay) = \beta_0 = \gamma_A(a) \). Therefore \( A = (\mu_A, \gamma_A) \) is an intuitionistic fuzzy interior ideal of \( S \). Obviously \( \mu_A^\alpha \beta_0 = M = \gamma_A^\alpha \beta_0 \).

Corollary 3.11. Let \( \chi_M \) be the characteristic function of an interior ideal \( M \) of \( S \). Then the IFS \( M = (\chi_M, \chi_M) \) is an intuitionistic fuzzy interior ideal of \( S \).

Theorem 3.12. If an IFS \( A = (\mu_A, \gamma_A) \) is an intuitionistic fuzzy interior ideal of \( S \), then

\[
\mu_A(x) := \sup \{ \alpha \in [0, 1] \mid x \in \mu_A^\alpha \},
\]

\[
\gamma_A(x) := \inf \{ \alpha \in [0, 1] \mid x \in \gamma_A^\alpha \},
\]

(3.10)

for all \( x \in S \).

Proof. Let \( \delta := \sup \{ \alpha \in [0, 1] \mid x \in \mu_A^\alpha \} \) and let \( \varepsilon > 0 \) be given. Then \( \delta - \varepsilon < \alpha \) for some \( \alpha \in [0, 1] \) such that \( x \in \mu_A^\alpha \). It follows that \( \delta - \varepsilon < \mu_A(x) \) so that \( \delta \leq \mu_A(x) \) since \( \varepsilon \) is arbitrary. We now show that \( \mu_A(x) \leq \delta \). Let \( \mu_A(x) = \beta \). Then \( x \in \mu_A^\beta \) and so

\[
\beta \in \{ \alpha \in [0, 1] \mid x \in \mu_A^\alpha \}.
\]

(3.11)

Hence \( \mu_A(x) = \beta \leq \sup \{ \alpha \in [0, 1] \mid x \in \mu_A^\alpha \} = \delta \). Therefore

\[
\mu_A(x) = \delta = \sup \{ \alpha \in [0, 1] \mid x \in \mu_A^\alpha \}.
\]

(3.12)

Now let \( \eta = \inf \{ \alpha \in [0, 1] \mid x \in \gamma_A^\alpha \} \). Then

\[
\inf \{ \alpha \in [0, 1] \mid x \in \gamma_A^\alpha \} < \eta + \varepsilon \quad \text{for any } \varepsilon < 0,
\]

(3.13)

and so \( \alpha < \eta + \varepsilon \) for some \( \alpha \in [0, 1] \) with \( x \in \gamma_A^\alpha \). Since \( \gamma_A(x) \leq \alpha \) and \( \varepsilon \) is arbitrary, it follows that \( \gamma_A(x) \leq \eta \). To prove \( \gamma_A(x) \geq \eta \), let \( \gamma_A(x) = \zeta \). Then \( x \in \gamma_A^\zeta \) and thus \( \zeta \in \{ \alpha \in [0, 1] \mid x \in \gamma_A^\alpha \} \). Hence

\[
\inf \{ \alpha \in [0, 1] \mid x \in \gamma_A^\alpha \} \leq \zeta, \quad \text{that is, } \eta \leq \zeta = \gamma_A(x).
\]

(3.14)
Consequently,
\[ y_A(x) = \eta = \inf \{ \alpha \in [0,1] \mid x \in y_{A,\alpha}^\gamma \}. \] (3.15)

This completes the proof. \( \square \)

**Theorem 3.13.** Let \( \{ C_\alpha \mid \alpha \in \Lambda \} \) be a collection of interior ideals of \( S \) such that

(i) \( S = \cup_{\alpha \in \Lambda} C_\alpha \),

(ii) \( \beta > \alpha \) if and only if \( C_\beta \subset C_\alpha \) for all \( \beta, \alpha \in \Lambda \).

Then an IFS \( A = (\mu_A, y_A) \) in \( S \) defined by

\[ \mu_A(x) := \sup \{ \alpha \in \Lambda \mid x \in C_\alpha \}, \]
\[ y_A(x) := \inf \{ \alpha \in \Lambda \mid x \in C_\alpha \}, \] (3.16)

for all \( x \in S \), is an intuitionistic fuzzy interior ideal of \( S \).

**Proof.** Following Theorem 3.9, it is sufficient to show that the nonempty level sets \( \mu_{A,\alpha} \) and \( y_{A,\alpha} \) are interior ideals of \( S \) for every \( \alpha \in [0,1] \). In order to prove that \( \mu_{A,\alpha} (\neq \emptyset) \) is an interior ideal, we have the following two cases:

(i) \( \alpha = \sup \{ \delta \in \Lambda \mid \delta < \alpha \} \) and

(ii) \( \alpha \neq \sup \{ \delta \in \Lambda \mid \delta < \alpha \} \).

Case (i) implies that

\[ x \in \mu_{A,\alpha} \iff x \in C_\delta \quad \forall \delta < \alpha \iff x \in \cap_{\delta < \alpha} C_\delta, \] (3.17)

so that \( \mu_{A,\alpha} = \cap_{\delta < \alpha} C_\delta \), which is an interior ideal of \( S \). For the case (iii), we claim that \( \mu_{A,\alpha} = \cup_{\delta \geq \alpha} C_\delta \). If \( x \in \cup_{\delta \geq \alpha} C_\delta \), then \( x \in C_\delta \) for some \( \delta \geq \alpha \). It follows that \( \mu_A(x) \geq \delta \geq \alpha \), so that \( x \in \mu_{A,\alpha} \). This proves that \( \mu_{A,\alpha} \) is an interior ideal of \( S \). Now assume that \( x \notin \cup_{\delta \geq \alpha} C_\delta \). Then \( x \notin C_\delta \) for all \( \delta \geq \alpha \). Since \( \alpha \neq \sup \{ \delta \in \Lambda \mid \delta < \alpha \} \), there exists \( \varepsilon > 0 \) such that \( (\alpha - \varepsilon, \alpha) \cap \Lambda = \emptyset \). Hence \( x \notin C_\delta \) for all \( \delta > \alpha - \varepsilon \), which means that if \( x \in C_\delta \) then \( \delta \leq \alpha - \varepsilon \). Thus \( \mu_A(x) = \alpha - \varepsilon < \alpha \), and so \( x \notin \mu_{A,\alpha} \). Therefore \( \mu_{A,\alpha} = \cup_{\delta \geq \alpha} C_\delta \), and thus \( \mu_{A,\alpha} = \cup_{\delta \geq \alpha} C_\delta \), which is an interior ideal of \( S \). Next we prove that \( y_{A,\alpha} (\neq \emptyset) \) is an interior ideal of \( S \) for all \( \alpha \in [0,1] \). We consider the following two cases:

(iii) \( \beta = \inf \{ \delta \in \Lambda \mid \beta < \delta \} \) and

(iv) \( \beta \neq \inf \{ \delta \in \Lambda \mid \beta < \delta \} \).

For the case (iii) we have

\[ x \in y_{A,\beta} \iff x \in C_\delta \quad \forall \beta < \delta \iff x \in \cap_{\beta \leq \delta} C_\delta, \] (3.18)

and hence \( y_{A,\beta} = \cap_{\beta \leq \delta} C_\delta \) which is an interior ideal of \( S \). For the case (iv), there exists \( \varepsilon > 0 \) such that \( (\beta, \beta + \varepsilon) \cap \Lambda = \emptyset \). We show that \( y_{A,\beta} = \cup_{\delta \leq \beta} C_\delta \). If \( x \in \cup_{\delta \leq \beta} C_\delta \), then \( x \in C_\delta \) for some \( \delta \geq \beta \). It follows that \( y_A(x) \leq \delta \leq \beta \) so that \( x \in y_{A,\beta} \). Hence \( \cup_{\delta \leq \beta} C_\delta \subset y_{A,\beta} \). Conversely, if \( x \notin \cup_{\delta \leq \beta} C_\delta \) then \( x \notin C_\delta \) for all \( \delta \leq \beta \), which implies that \( x \notin C_\delta \) for all \( \delta < \beta + \varepsilon \), that is, if \( x \in C_\delta \) then \( \delta \geq \beta + \varepsilon \). Thus \( y_A(x) \geq \beta + \varepsilon > \beta \), that is, \( x \notin y_{A,\beta} \). Therefore \( y_{A,\beta} \subset \cup_{\delta \leq \beta} C_\delta \) and consequently \( y_{A,\beta} = \cup_{\delta \leq \beta} C_\delta \) which is an interior ideal of \( S \). This completes the proof. \( \square \)

**Theorem 3.14.** An IFS \( A = (\mu_A, y_A) \) is an intuitionistic fuzzy interior ideal of \( S \) if and only if the fuzzy sets \( \mu_A \) and \( y_A \) are fuzzy interior ideals of \( S \).
Proof. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy interior ideal of $S$. Then clearly $\mu_A$ is a fuzzy interior ideal of $S$. Let $x, a, y \in S$. Then

$$\tilde{\gamma}_A(xy) = 1 - \gamma_A(xy) \geq 1 - \gamma_A(x) \land \gamma_A(y) = (1 - \gamma_A(x)) \land (1 - \gamma_A(y)) = \tilde{\gamma}_A(x) \land \tilde{\gamma}_A(y), \quad (3.19)$$

$$\tilde{\gamma}_A(xay) = 1 - \gamma_A(xay) \geq 1 - \gamma_A(a) = \tilde{\gamma}_A(a).$$

Hence $\tilde{\gamma}_A$ is a fuzzy interior ideal of $S$.

Conversely, suppose that $\mu_A$ and $\tilde{\gamma}_A$ are fuzzy interior ideals of $S$. Let $a, x, y \in S$. Then

$$1 - \gamma_A(xy) = \tilde{\gamma}_A(xy) \geq \tilde{\gamma}_A(x) \land \tilde{\gamma}_A(y) = (1 - \gamma_A(x)) \land (1 - \gamma_A(y)) = 1 - \gamma_A(x) \lor \gamma_A(y),$$

$$1 - \gamma_A(xay) = \tilde{\gamma}_A(xay) \geq \tilde{\gamma}_A(a) = 1 - \gamma_A(a),$$

which imply that $\gamma_A(xy) \leq \gamma_A(x) \lor \gamma_A(y)$ and $\gamma_A(xay) \leq \gamma_A(a)$. This completes the proof.

Corollary 3.15. An IFS $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy interior ideal of $S$ if and only if $\Box A = (\mu_A, \bar{\mu}_A)$ and $\Diamond A = (\tilde{\gamma}_A, \gamma_A)$ are intuitionistic fuzzy interior ideals of $S$.

Proof. The proof is straightforward by Theorem 3.14.

Theorem 3.16. Let $f : S \to T$ be a homomorphism of semigroups. If $B = (\mu_B, \gamma_B)$ is an intuitionistic fuzzy interior ideal of $T$, then the preimage $f^{-1}(B)$ under $f$, denoted by $f^{-1}(B)$, is an IFS in $S$ defined by

$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)), \quad \text{where } f^{-1}(\mu_B) = \mu_B(f). \quad (3.21)$$

Proof. Assume that $B = (\mu_B, \gamma_B)$ is an intuitionistic fuzzy interior ideal of $T$ and let $x, y \in S$. Then

$$f^{-1}(\mu_B)(xy) = \mu_B(f(xy))$$

$$= \mu_B(f(x)f(y))$$

$$\geq \mu_B(f(x)) \land \mu_B(f(y))$$

$$= f^{-1}(\mu_B(x)) \land f^{-1}(\mu_B(y)), \quad (3.22)$$

$$f^{-1}(\gamma_B)(xy) = \gamma_B(f(xy))$$

$$= \gamma_B(f(x)f(y))$$

$$\leq \gamma_B(f(x)) \lor \gamma_B(f(y))$$

$$= f^{-1}(\gamma_B(x)) \lor f^{-1}(\gamma_B(y)).$$
Hence \( f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)) \) is an intuitionistic fuzzy subsemigroup of \( S \). For any \( a, x, y \in S \), we have

\[
\begin{align*}
    f^{-1}(\mu_B)(xay) &= \mu_B(f(xay)) \\
    &= \mu_B(f(x)f(a)f(y)) \\
    &\geq \mu_B(f(a)) \\
    &= f^{-1}(\mu_B(a)),
\end{align*}
\]

\[
\begin{align*}
    f^{-1}(\gamma_B)(xay) &= \gamma_B(f(xay)) \\
    &= \gamma_B(f(x)f(a)f(y)) \\
    &\leq \gamma_B(f(a)) \\
    &= f^{-1}(\gamma_B(a)).
\end{align*}
\]

Therefore \( f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)) \) is an intuitionistic fuzzy interior ideal of \( S \).

\[\square\]

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