ON CONNECTEDNESS IN INTUITIONISTIC FUZZY SPECIAL TOPOLOGICAL SPACES

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ABSTRACT. The aim of this paper is to construct the basic concepts related to connectedness in intuitionistic fuzzy special topological spaces. Here we introduce the concepts of $C_5$-connectedness, connectedness, $C_6$-connectedness, $C_M$-connectedness, strong connectedness, super connectedness, $C_i$-connectedness ($i=1,2,3,4$), and obtain several preservation properties and some characterizations concerning connectedness in these spaces.

KEY WORDS AND PHRASES. Intuitionistic fuzzy special set; intuitionistic fuzzy special topology, intuitionistic fuzzy special topological space, continuity; $C_5$-connectedness; connectedness; $C_6$-connectedness; $C_M$-connectedness; strong connectedness; super connectedness; $C_i$-connectedness ($i=1,2,3,4$).

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1. INTRODUCTION

After the introduction of the concept of fuzzy sets by Zadeh [1] several researches were conducted on the generalizations of the notion of fuzzy set. The idea of intuitionistic fuzzy set was first published by Krassimir Atanassov [2] and many works by the same author appeared in the literature (see Atanassov [2,3]) Later this concept is used to define intuitionistic fuzzy special sets by Çoker [4] and intuitionistic fuzzy topological spaces are introduced by Çoker [5], Coker-Es [6]. In this direction some preliminary concepts are also defined by Coğkun-Çoker[7]. Here we shall give the classical version of this kind of fuzzy topological space in the framework of connectedness;
especially, we shall make use of several types of fuzzy connectedness in intuitionistic fuzzy topological spaces in Turanli-Coker [8].

2. PRELIMINARIES

First we shall present the fundamental definitions. The following one is obviously inspired by K. Atanassov [2,3]:

**DEFINITION 2.1.** (see Çoker [4]) Let \( X \) be a nonempty fixed set. An intuitionistic fuzzy special set (IFSS for short) \( A \) is an object having the form \( A = \langle x, A_1, A_2 \rangle \), where \( A_1 \) and \( A_2 \) are subsets of \( X \) satisfying \( A_1 \cap A_2 = \emptyset \). The set \( A_1 \) is called the set of members of \( A \), while \( A_2 \) is called the set of nonmembers of \( A \).

Obviously every set \( A \) on a nonempty set \( X \) is obviously an IFSS having the form \( \langle x, A, A \rangle \). One can define several relations and operations between IFSS's as follows:

**DEFINITION 2.2.** (see Çoker [4,5]) Let \( X \) be a nonempty set, and the IFSS's \( A \) and \( B \) be in the form \( A = \langle x, A_1, A_2 \rangle \), \( B = \langle x, B_1, B_2 \rangle \), respectively. Furthermore, let \( \langle A_i \rangle \) be an arbitrary family of IFSS's in \( X \), where \( A = \langle x, A_1^{(1)}, A_2^{(2)} \rangle \). Then

\[
\begin{align*}
(a) & \quad A \subseteq B \text{ iff } A_1 \subseteq B_1 \text{ and } A_2 \subseteq B_2 ; \\
(b) & \quad A = B \text{ iff } A \subseteq B \text{ and } B \subseteq A ; \\
(c) & \quad \overline{A} = \langle x, A_1, A_1 \rangle ; \\
(d) & \quad \bigcap A = \langle x, A_1, A_1 \rangle , \\
(e) & \quad \langle A_i \rangle = \langle x, A_1 \cup A_1 \rangle ; \\
(f) & \quad \bigcup A = \langle x, A_1 \cup A_2 \rangle , \\
(g) & \quad \cap A_1 = \langle x, A_1 \cap A_1 \rangle ; \\
(h) & \quad \ominus = \langle x, \emptyset, X \rangle \text{ and } X = \langle x, X, \emptyset \rangle .
\end{align*}
\]

We shall define the image and preimage of IFSS's. Let \( X \) and \( Y \) be two nonempty sets and \( f \colon X \to Y \) a function.

**DEFINITION 2.3.** (see Çoker [4,5]) (a) If \( B = \langle y, B_1, B_2 \rangle \) is an IFSS in \( Y \), then the preimage of \( B \) under \( f \), denoted by \( f^{-1}(B) \), is the IFSS in \( X \) defined by \( f^{-1}(B) = \langle x, f^{-1}(B_1), f^{-1}(B_2) \rangle \). 

(b) If \( A = \langle x, A_1, A_2 \rangle \) is an IS in \( X \), then the image of \( A \) under \( f \), denoted by \( f(A) \), is the IFSS in \( Y \) defined by \( f(A) = \langle y, f(A_1), f(A_2) \rangle \), where \( f(A_1) = (f(A))_1 \).

**COROLLARY 2.1.** Let \( A, A, (i \in J) \) be IFSS's in \( X, B, B, (j \in K) \) IFSS's in \( Y \) and \( f : X \to Y \) a function. Then

\[
\begin{align*}
(a) & \quad A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2) ; \\
(b) & \quad B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2) \\
(c) & \quad A \subseteq f^{-1}(f(A)) \text{ and if } f \text{ is injective, then } A = f^{-1}(f(A)) . \\
(d) & \quad f(f^{-1}(B)) \subseteq B, \text{ and if } f \text{ is surjective, then } f(f^{-1}(B)) = B \\
(e) & \quad f^{-1}(\cup B_j) = \cup f^{-1}(B_j) \quad \text{ and } f^{-1}(\cap B_j) = \cap f^{-1}(B_j) ; \\
(f) & \quad f^{-1}(\cap A_i) = \cap f^{-1}(A_i) \text{ and if } f \text{ is injective, then } f(\cap A_i) = \cap f(A_i) . \\
(i) & \quad f^{-1}(Y) = \emptyset \quad \text{ and if } f \text{ is surjective, then } f(\emptyset) = \emptyset \\
(k) & \quad f(X) = Y \text{ if } f \text{ is surjective.} \\
(l) & \quad f(\emptyset) = \emptyset \\
(m) & \quad \text{If } f \text{ is surjective, then } f(\emptyset) = f(\emptyset) , \text{ and if, furthermore, } f \text{ is injective, we have } f(\emptyset) = f(\emptyset) . \\
(n) & \quad f^{-1}(\emptyset) = f^{-1}(\emptyset) \\
\end{align*}
\]
DEFINITION 2.4 (see Coker [5,9], Coker-Es [6]) An intuitionistic fuzzy special topology (IFST for short) on a nonempty set $X$ is a family $\tau$ of IFSS's in $X$ containing $\emptyset$, $X$, and closed under finite infima and arbitrary suprema. In this case the pair $(X, \tau)$ is called an intuitionistic fuzzy special topological space (IFSTS for short) and any IFSS in $\tau$ is known as an intuitionistic fuzzy special open set (IFOS for short) in $X$.

Any topological space can be obviously treated as an IFSTS in a usual manner.

PROPOSITION 2.1. Let $(X, \tau)$ be an IFSTS on $X$. Then, we can also construct several IFSTS's on $X$ in the following way.

(a) $\tau_{0.1} = \{ G : G \in \tau \}$,  
(b) $\tau_{0.2} = \{ \alpha : G \in \tau \}$.

REMARK 2.1 Let $(X, \tau)$ be an IFSTS $\tau_1 = \{ G_1 : G_1 < x, G_2 \geq \tau \}$ is a topological space on $X$ $\tau_2 = \{ G_2 : G_2 < x, G_2 \geq \tau \}$ is the family of all closed sets of the topological space $(\tau_2 = \{ G_2 : G_2 < x, G_2 \geq \tau \}$ on $X$.

The complement $\overline{A}$ of an IFOS $A$ in an IFSTS $(X, \tau)$ is called an intuitionistic fuzzy special closed set (IFSCS for short) in $X$, and the interior and closure of an IFSS $A$ are defined by

$\text{cl}(A) = \bigcap \{ K : K \text{ is an IFSCS in } X \text{ and } A \subseteq K \}$,

$\text{int}(A) = \bigcup \{ G : G \text{ is an IFOS in } X \text{ and } G \supseteq A \}$

DEFINITION 2.5. Let $(X, \tau)$ be an IFSTS on $X$ If $A = \text{int}(\text{cl}(A))$, then $A$ is called an intuitionistic fuzzy special regular open set in $X$.

DEFINITION 2.6. Let $(X, \tau)$ and $(Y, \psi)$ be two IFSTS's and let $f : X \rightarrow Y$ be a function. Then $f$ is said to be continuous if the preimage of each IFSS in $\psi$ is an IFSS in $\tau$.

Here we obtain some characterizations of continuity.

PROPOSITION 2.2 The following are equivalent to each other:

(a) $f : (X, \tau) \rightarrow (Y, \psi)$ is continuous.

(b) The preimage of each IFSCS in $Y$ is an IFSCS in $X$.

(c) $f^{-1}(\text{int}(B)) \subseteq \text{int}(f^{-1}(B))$ for each IFSS $B$ in $Y$.

(d) $\text{cl}(f^{-1}(B)) \subseteq (f^{-1}(\text{cl}(B))$ for each IFSS $B$ in $Y$.

3. TYPES OF CONNECTEDNESS IN INTUITIONISTIC FUZZY SPECIAL TOPOLOGICAL SPACES

Throughout this section $(X, \tau)$ and $(Y, \psi)$ will always denote IFSTS's. We shall define several types of connectedness in IFSTS's.

DEFINITION 3.1. (see Chaudhuri-Das [10], Turanli-Coker [8])

(a) $X$ is called $C_3$-disconnected, if there exists an IFSS $A$ which is both intuitionistic fuzzy special open and intuitionistic fuzzy special closed, such that $\emptyset \neq A \neq X$.

(b) $X$ is called $C_3$-connected, if $X$ is not $C_3$-disconnected.

(c) $X$ is called disconnected, if there exist IFOS's $A \neq \emptyset$ and $B \neq \emptyset$ such that $A \cup B = X$ and $A \cap B = \emptyset$. 
(d) X is called connected, if X is not disconnected.

**Proposition 3.1.** \( C_3 \)-connectedness implies connectedness.

**Proof.** Suppose that there exist nonempty IFSOS's A and B such that \( A \cup B = X \), \( A \cap B = \emptyset \), from which we get \( A \cup B_1 = X \), \( A_2 \cap B_2 = \emptyset \), \( A_1 \cap B_1 = \emptyset \), \( A_2 \cup B_2 = X \), in other words, \( A = B \). Hence A is intuitionistic fuzzy special clopen, i.e. \( (X, \tau) \) is \( C_3 \)-disconnected.

**Counterexample 3.1.** Consider the IFTS \( \tau \) on \( X = \{a, b, c, d\} \), where \( \tau = \{ \emptyset, X, A_1, A_2, A_3, A_4 \} \), \( A_1 = \langle x, \{a\}, \{b, c\} \rangle, A_2 = \langle x, \{b, c\}, \{a\} \rangle, A_3 = \langle x, \emptyset, \{a, b, c\} \rangle, A_4 = \langle x, \{a, b, c\}, \emptyset \rangle \) \( (X, \tau) \) is connected, but not \( C_3 \)-connected (namely, \( A_4 \) is intuitionistic fuzzy special clopen in \( X \)).

**Proposition 3.2.** Let \( f : (X, \tau) \to (Y, \psi) \) be a continuous surjection. If \( X \) is connected, then so is \( Y \).

**Proof.** Assume that \( Y \) is disconnected. Thus there exist IFSOS's \( C \neq \emptyset, D \neq \emptyset \) in \( Y \) such that \( C \cup D = Y, C \cap D = \emptyset \). Now we see that \( A = f^{-1}(C), B = f^{-1}(D) \) are IFSOS's in \( X \), since \( f \) is continuous. From \( C \neq \emptyset \), we get \( A = f^{-1}(C) \neq \emptyset \) (if \( f^{-1}(C) = \emptyset \), then \( C = f(f(\emptyset)) = \emptyset \), which is a contradiction.) Similarly, we obtain \( B \neq \emptyset \). Now \( C \cup D = Y \Rightarrow f^{-1}(C) \cup f^{-1}(D) = f^{-1}(Y) = X \Rightarrow A \cup B = X, C \cap D = \emptyset \Rightarrow f^{-1}(C) \cap f^{-1}(D) = f^{-1}(\emptyset) = \emptyset \Rightarrow A \cap B = \emptyset \). But this is a contradiction to our hypothesis, thus \( Y \) is connected.

**Proposition 3.3.** If \( (X, \tau) \) is disconnected, then so are the IFTS's \( (X, \tau_{01}) \) and \( (X, \tau_{02}) \).

**Proof.** Let there exist IFSOS's \( A \neq \emptyset, B \neq \emptyset \) such that \( A \cap B = X \), \( A \cup B = \emptyset \). In this case we obtain \( X = \emptyset, \emptyset = \emptyset \), \( A \cup B = \emptyset, A \cap B = \emptyset \). In this case we obtain \( X = \emptyset, \emptyset = \emptyset \), \( A \cup B = \emptyset, A \cap B = \emptyset \), which is a contradiction.

**Proposition 3.4.** \( (X, \tau) \) is \( C_3 \)-connected iff there exist no nonempty IFSOS's A and B in \( X \) such that \( A = B \).

**Proof.** (\( \Rightarrow \)) Suppose that A and B are IFSOS's in \( X \) such that \( A \neq B \) and \( A = B \). Since \( A = B \), B is an IFSCS, and \( A \neq B \Rightarrow B \neq X \). But this is a contradiction to the fact that \( X \) is \( C_3 \)-connected.

(\( \Leftarrow \)) Let A be both an IFSOS and IFSCS such that \( \emptyset \neq A \neq X \). Now take \( B = \overline{A} \). In this case B is an IFSOS and \( A \neq X \Rightarrow B = \overline{A} \neq \emptyset \), which is a contradiction.

**Proposition 3.5.** \( (X, \tau) \) is \( C_3 \)-connected iff there exist no nonempty IFSS's A and B in \( X \) such that \( B = \overline{A}, \overline{B} = \overline{A}, A = \overline{B} \).

**Proof.** (\( \Rightarrow \)) Assume that there exist IFSS's A and B such that \( A \neq B \) and \( B = \overline{A}, A = \overline{B} \). Hence \( \overline{A} \) and \( \overline{B} \) are IFSOS's in \( X \), A and B are IFSOS's in \( X \), which is a contradiction (
Let $A$ be both an IFSOS and IFSCS in $X$ such that $\varnothing \neq A \neq X$. Taking $B = \overline{A}$, we obtain a contradiction.

Here we generalize the concepts of $C_s$-connectedness and $C_M$-connectedness given by Chaudhuri - Das [10] to the intuitionistic case:

**Lemma 3.1.** (a) $A \cap B = \varnothing \Rightarrow A \subseteq \overline{B}$, (b) $A \cap \overline{B} \Rightarrow A \cap B = \varnothing$

**Definition 3.2.** Let $A$ and $B$ be nonzero IFSS's in $(X, \tau)$. $A$ and $B$ are said to be weakly separated, if $\text{cl}(A) \subseteq \overline{B}$ and $\text{cl}(B) \subseteq \overline{A}$; and $q$-separated, if $\text{cl}(A) \cap B = \varnothing = A \cap \text{cl}(B)$.

**Definition 3.3.** (see Turanli-Coker [8]) An IFSTS $(X, \tau)$ is said to be $C_s$-disconnected, if there exist weakly separated nonzero IFSS's $A$ and $B$ in $(X, \tau)$ such that $X = A \cup B$.

(b) $(X, \tau)$ is called $C_s$-connected, if $(X, \tau)$ is not $C_s$-disconnected.

(c) $X$ is said to be $C_M$-disconnected, if there exist $q$-separated nonzero IFSS's $A$ and $B$ in $X$ such that $X = A \cup B$.

(d) $X$ is called $C_M$-connected, if $X$ is not $C_M$-disconnected.

Let us give the connection between these two types of connectedness in IFSTS's:

**Corollary 3.1.** If the IFSTS $X$ is $C_s$-connected, then $X$ is also $C_M$-connected.

**Definition 3.4.** (see Turanli-Coker [8]) An IFSTS $(X, \tau)$ is said to be strongly connected, if there exist no nonempty IFSCS's $A$ and $B$ in $X$ such that $A \cap B = \varnothing$.

**Proposition 3.6.** $X$ is strongly connected iff there exist no IFSOS's $A$ and $B$ in $X$ such that $A \neq X = B$ and $A \cup B = X$.

**Proof.** ($\Rightarrow$) Let $A$ and $B$ be IFSOS's in $X$ such that $A \neq X = B$ and $A \cup B = X$. If we take $C = \overline{A}$ and $D = \overline{B}$, then $C$ and $D$ become IFSCS's in $X$ and $C \neq D = \varnothing$, a contradiction.

($\Leftarrow$) Use a similar technique as above.

**Proposition 3.7.** Let $f : (X, \tau) \to (Y, \psi)$ be a continuous surjection. If $X$ is strongly connected, then so is $Y$.

**Proof.** Suppose that $Y$ is not strongly connected. In this case there exist IFSCS's $C$ and $D$ in $Y$ such that $C \neq D = \varnothing$, $C \cap D = \varnothing$. Since $f$ is continuous, $f^{-1}(C)$ and $f^{-1}(D)$ are IFSCS's in $X$, and $f^{-1}(C) \cap f^{-1}(D) = \varnothing$, $f^{-1}(C) \neq D$, $f^{-1}(D) \neq C$. (If $f^{-1}(C) = \varnothing$, then $f(f^{-1}(C)) = C \Rightarrow f(\varnothing) = C \Rightarrow \varnothing = C$, a contradiction.) But this is a contradiction, hence $Y$ is strongly connected, too.

Strong connectedness does not imply $C_s$-connectedness, and the same is true for IFSTS converse, i.e. $C_s$ connectedness does not imply strong connectedness. For this purpose see the following counterexamples:

**Counterexamples 3.2.** Let $X = \{a, b, c, d\}$ (a) If $\tau = (\varnothing, X, A_1, A_2, A_3, A_4)$, where $A_1 = \langle x, \{b, c\}, \{d\} \rangle$, $A_2 = \langle x, \{d\}, \{b, c\} \rangle$, $A_3 = \langle x, \varnothing, \{b, c, d\} \rangle$, $A_4 = \langle x, \{b, c, d\}, \varnothing \rangle$, then the IFSTS $(X, \tau)$ is strongly connected, but not $C_s$-connected.
(b) If $\tau=\{\emptyset, X, A_1, A_2, A_3, A_4, A_5\}$, where $A_1 =^{<}_{<} x, \{b, c\}, \{d\}$, $A_2 =^{<}_{<} x, \{a\}, \{c\}$, $A_3 =^{<}_{<} x, \{a, d\}, \{c\}$, $A_4 =^{<}_{<} x, \{a, b, c\}, \emptyset$, $A_5 =^{<}_{<} x, \emptyset, \{c, d\}$, then the IFSTS $(X, \tau)$ is $C_s$-connected, but not strongly connected.

**DEFINITION 3.5.** (see Turanli-Coker [8]) (a) If there exists an intuitionistic fuzzy special regular open set $A$ in $X$ such that $\emptyset \neq A \neq X$, then $X$ is called super disconnected.

(b) $X$ is called super connected, if $X$ is not super disconnected.

Now we give some characterizations of super connectedness:

**PROPOSITION 3.8.** The following assertions are equivalent:

(a) $X$ is super connected.

(b) For each IFSOS $A \neq \emptyset$ in $X$ we have $\text{cl}(A) = X$

(c) For each IFSCS $A \neq \emptyset$ in $X$ we have $\text{int}(A) = \emptyset$

(d) There exist no IFSOS's $A$ and $B$ in $X$ such that $A \neq \emptyset \neq B$, $A \subseteq \overline{B}$.

(e) There exist no IFSOS's $A$ and $B$ in $X$ such that $A \neq \emptyset \neq B$, $B = \overline{\text{cl}(A)}$, $A = \overline{\text{cl}(B)}$

(f) There exist no IFSCS's $A$ and $B$ in $X$ such that $A \neq \emptyset \neq B$, $B = \overline{\text{int}(A)}$, $A = \overline{\text{int}(B)}$

**PROOF.** (a) $\Rightarrow$ (b) : Assume that there exists an IFSOS $A \neq \emptyset$ such that $\text{cl}(A) \neq X$. Now take $B = \overline{\text{int}(\text{cl}(A))}$. Then $B$ is a proper intuitionistic fuzzy special regular open set in $X$, and this is in contradiction with the super connectedness of $X$.

(b) $\Rightarrow$ (c) : Let $A \neq \emptyset$ be an IFSCS in $X$. If we take $B = \overline{A}$, then $B$ is an IFSOS in $X$ and $B = \emptyset$.

Hence $\text{cl}(B) = \emptyset = \overline{\text{cl}(B)} = \emptyset = \text{int}(\text{cl}(B)) = \text{int}(A) = \emptyset$ follows.

(c) $\Rightarrow$ (d) : Let $A$ and $B$ be IFSOS's in $X$ such that $A \neq \emptyset \neq B$ and $A \subseteq \overline{B}$. Since $\overline{B}$ is an IFCS in $X$ and $B \neq \emptyset \Rightarrow \overline{B} \neq \emptyset$, we obtain $\text{int}(\overline{B}) = \emptyset$. But, from $A \subseteq \overline{B}$, we see that $\emptyset \neq A = \text{int}(A) \subseteq \text{int}(\overline{B}) = \emptyset$, which is a contradiction.

(d) $\Rightarrow$ (a) : Let $\emptyset \neq A \neq X$ be an intuitionistic fuzzy special regular open set in $X$. If we take $B = \overline{\text{cl}(A)}$, we get $B \neq \emptyset$. (Because, otherwise we have $B = \emptyset \Rightarrow \text{cl}(A) = \emptyset \Rightarrow \text{cl}(A) = X \Rightarrow A = \text{int}(\text{cl}(A)) = \text{int}(X) = \emptyset$, but the last result contradicts the fact $A \neq \emptyset$.) We also have $A \subseteq \overline{B}$, and this is a contradiction, too.

(a) $\Rightarrow$ (e) : Let $A$ and $B$ be IFSOS's in $X$ such that $A \neq \emptyset \neq X$ and $B = \overline{\text{cl}(A)}$, $A = \overline{\text{cl}(B)}$. Now we have $\text{int}(\text{cl}(A)) = \text{int}(\overline{B}) = \text{cl}(B) = A$ and $A \neq \emptyset$, $A \neq X$. (If not, i.e. if $A = \emptyset$, then $X = \overline{\text{cl}(B)} \Rightarrow \emptyset = \text{cl}(B) \Rightarrow B = \emptyset$.) But this is a contradiction.

(e) $\Rightarrow$ (a) : Let $A$ be an IFSOS in $X$ such that $A = \text{int}(\text{cl}(A))$, $\emptyset \neq A \neq X$. Now take $B = \overline{\text{cl}(A)}$. In this case we get $B \neq \emptyset$ and $B$ is an IFSOS in $X$ and $B = \overline{\text{cl}(A)}$ and $\overline{\text{cl}(B)} = \text{cl}(\overline{\text{cl}(A)}) = \text{int}(\text{cl}(A)) = \text{int}(\text{cl}(A)) = A$, which is a contradiction.
Let A and B IFSCS's in X such that A$\subseteq$ X $\nsubseteq$B, B=int(A), A=int(B) Taking C=$\overline{A}$ and D=$\overline{B}$, C and D become IFSOS's in X and C$\nsubseteq$C$\nsubseteq$D. cl(C) = $\overline{\text{int}(A)}$ = int(A)=int(A)=$\overline{B}$=D, and similarly cl(D)=C. But this is an obvious contradiction.

(f) $\Rightarrow$ (e): One can use a similar technique as in (e) $\Rightarrow$ (f).

**PROPOSITION 3.9.** Super connectedness implies C$_5$-connectedness.

**PROOF.** Obvious.

But the reverse implication to Proposition 3.9 does not hold in general. **COUNTEREXAMPLE 3.3.** Let X={a,b,c,d} and the IFST $\tau$={ ,X,A$_1$,A$_2$,A$_3$,A$_4$} on X, where A$_1$ =<x,{a},{c,d}>, A$_2$ =<x,{d},{a,c}>, A$_3$ =<x,{a,d}>, A$_4$ =<x, ,{a,c,d}$. Then the IFSTS (X,$\tau$) is C$_5$ -connected, but not super connected.

**PROPOSITION 3.10.** Let f:(X,$\tau$) $\rightarrow$ (Y,$\psi$) be a continuous surjection. If X is super connected, then so is Y.

**PROOF.** Suppose that Y is super disconnected. In this case there exist IFSOS's C and D in Y such that C$\nsubseteq$D $\nsubseteq$D. Since f is continuous, f$^{-1}$(C) and f$^{-1}$(D) are IFSOS's in X, and C$\subseteq$D $\Rightarrow$ f$^{-1}$(C)$\subseteq$f$^{-1}$(D), f$^{-1}$(C)$\nsubseteq$D $\nsubseteq$f$^{-1}$(D), which means that X is super disconnected.

Now we shall summarize the interrelations between several types of connectedness in IFSTS's.

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    super connectedness   C$_5$-connectedness
   ↓                     ↓
C$_5$-connectedness   C$_M$-connectedness
   ↓
connectedness
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Here we generalize the idea of fuzzy C$_i$-connectedness in fuzzy topological spaces and in intuitionistic fuzzy topological spaces (see Ajmal-Kohli [11], Chaudhuri-Das [10] and Turanli-Çoker [8] to the intuitionistic case:

**DEFINITION 3.6.** Let N be an IFSS in (X,$\tau$)

(a) If there exist IFSOS's M and W in X satisfying the following properties, then N is called C$_i$-disconnected (i=1,2,3,4):

C$_i$: N$\subseteq$M$\cap$W, M$\cap$W$\subseteq$N, N$\nsubseteq$M$\nsubseteq$N$\nsubseteq$N$\nsubseteq$M$\nsubseteq$W$\nsubseteq$W$\nsubseteq$N, C$_2$: N$\subseteq$M$\cap$W, N$\cap$M$\cap$W= , N$\cap$W$\nsubseteq$N$\nsubseteq$M$\nsubseteq$W$\nsubseteq$W$\nsubseteq$N.

C$_3$: M$\subseteq$M$\cap$W, M$\cap$W$\subseteq$N, M$\subseteq$N$\subseteq$N, M$\subseteq$N$\subseteq$N$\subseteq$N$\subseteq$N, C$_4$: N$\subseteq$M$\cap$W, N$\cap$M$\cap$W= , M$\subseteq$M$\subseteq$N, M$\subseteq$N.

(b) N is said to be C$_i$-connected (i=1,2,3,4), if N is not C$_i$-disconnected (i=1,2,3,4).

Obviously, one can obtain the following implications between several types of C$_i$-connectedness (i=1,2,3,4):

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C$_1$-connectedness $\rightarrow$ C$_2$-connectedness
      ↓
C$_3$-connectedness $\rightarrow$ C$_4$-connectedness
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None of these implications are reversible, as the following counterexamples state:
COUNTEREXAMPLES 3.4. Consider the IFST $\tau$ on $X=\{a, b\}$, where

$\tau=\{\emptyset, X, A_1, A_2, A_3, A_4, A_5, A_6, A_7\}, A_1 =\langle x, \{a\} \rangle, A_2 =\langle x, \emptyset \rangle, A_3 =\langle x, \emptyset, \{a\} \rangle, A_4 =\langle x, \emptyset \rangle, A_5 =\langle x, \{a\} \rangle, A_6 =\langle x, \{a\}, \{b\} \rangle, A_7 =\langle x, \{b\} \rangle$ and take the IFSS $N =\langle x, \emptyset, \{a\} \rangle$ in $X$.

(a) $N$ is $C_2$-connected, but not $C_1$-connected. [Namely, $A_2$ and $A_3$ do satisfy the properties in $C_1$.] 
(b) $N$ is $C_3$-connected, but not $C_1$-connected.

COUNTEREXAMPLE 3.5. Consider the IFST $\tau$ on $X=\{a, b, c, d\}$, where $\tau=\{\emptyset, X, A_1, A_2, A_3, A_4\}, A_1 =\langle x, \{a\} \rangle, A_2 =\langle x, \emptyset \rangle, A_3 =\langle x, \emptyset \rangle, A_4 =\langle x, \emptyset \rangle$.

The IFSS $N =\langle x, \{a\} \rangle$ in $X$ is $C_3$-connected, but not $C_2$-connected [Namely, $A_1$ and $A_2$ do satisfy the properties in $C_1$.]

COUNTEREXAMPLE 3.6. Consider the IFST $\tau$ on $X=\{a, b, c\}$, where $\tau=\{\emptyset, X, A_1, A_2, A_3\}, A_1 =\langle x, \{a\} \rangle, A_2 =\langle x, \emptyset \rangle, A_3 =\langle x, \emptyset \rangle$.

The IFSS $N =\langle x, \{a\} \rangle$ in $X$ is $C_4$-connected, but not $C_2$-connected. [Namely, $A_1$ and $A_2$ do satisfy the properties in $C_2$.]

REFERENCES


