ERRATUM TO “HYPERFINITE AND STANDARD UNIFICATIONS FOR PHYSICAL THEORIES”

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This corrects the major theorem on product consequence operators.

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In [1], Definition 5.2, and Theorem 5.3 and its proof are stated incorrectly. The following is the correct definition, theorem, and proof.

Definition 5.2. Suppose one has a nonempty finite set \( \mathcal{C} = \{C_1, \ldots, C_m\} \) of general consequence operators, each defined on a nonempty language \( L_i, 1 \leq i \leq m \). Define the operator \( \Pi C_m \) as follows: for any \( X \subset L_1 \times \cdots \times L_m \), using the projection \( pr_i, 1 \leq i \leq m \), define \( \Pi C_m(X) = C_1(pr_1(X)) \times \cdots \times C_m(pr_m(X)) \).

Theorem 5.3. The operator \( \Pi C_m \) defined on the subsets of \( L_1 \times \cdots \times L_m \) is a general consequence operator and if, at least, one member of \( \mathcal{C} \) is axiomless, then \( \Pi C_m \) is axiomless. If each member of \( \mathcal{C} \) is finitary and axiomless, then \( \Pi C_m \) is finitary.

Proof. (a) Let \( X \subset L_1 \times \cdots \times L_m \). Then for each \( i, 1 \leq i \leq m \), \( pr_i(X) \subset C_i(pr_i(X)) \subset L_i \). But, \( X \subset pr_1(X) \times \cdots \times pr_m(X) \subset C_1(pr_1(X)) \times \cdots \times C_m(pr_m(X)) = \Pi C_m(X) \subset L_1 \times \cdots \times L_m \). Suppose that \( X \neq \emptyset \). Then \( \emptyset \neq \Pi C_m(X) = C_1(pr_1(X)) \times \cdots \times C_m(pr_m(X)) \subset L_1 \times \cdots \times L_m \). Hence, \( \emptyset \neq pr_i(\Pi C_m(X)) = C_i(pr_i(X)), 1 \leq i \leq m \), implies that \( C_i(pr_i(\Pi C_m(X))) = C_i(pr_i(X)), 1 \leq i \leq m \). Hence, \( \Pi C_m(\Pi C_m(X)) = \Pi C_m(X) \).

Let \( X = \emptyset \) and assume that no member of \( \mathcal{C} \) is axiomless. Then each \( pr_i(X) = \emptyset \). But, each \( C_i(pr_i(X)) \neq \emptyset \) implies that \( \Pi C_m(X) \neq \emptyset \). By the previous method, it follows, in this case, that \( \Pi C_m(\Pi C_m(X)) = \Pi C_m(X) \).

Now suppose that there is some \( j \) such that \( C_j \) is axiomless. Hence, \( C_j(pr_j(X)) = \emptyset \) implies that \( \Pi C_m(X) = C_1(pr_1(X)) \times \cdots \times C_m(pr_m(X)) = \emptyset \), which implies that \( C_j(pr_j(\Pi C_m(X))) = \emptyset \). Consequently, \( C_j(pr_j(\Pi C_m(X))) \times \cdots \times C_m(pr_m(\Pi C_m(X)))) = \emptyset \). Thus, \( \Pi C_m(\Pi C_m(X)) = \emptyset \) and axiom (1) holds. Also in the case where at least one member of \( \mathcal{C} \) is axiomless, then \( \Pi C_m \) is axiomless.

(b) Let \( X \subset Y \subset L_1 \times \cdots \times L_m \). For each \( i, 1 \leq i \leq m \), \( pr_i(X) \subset pr_i(Y) \), whether \( pr_i(X) \) is the empty set or not. Hence, \( C_i(pr_i(X)) \subset C_i(pr_i(Y)) \).

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\[ C_1(pr_1(X)) \times \cdots \times C_m(pr_m(X)) \subset C_1(pr_1(Y)) \times \cdots \times C_m(pr_m(Y)) = \Pi C_m(Y) \] and axiom (2) holds. Thus, \( \Pi C_m \) is, at least, a general consequence operator.

(c) Assume that each member of \( \mathcal{C} \) is finitary and axiomless and let \( x \in \Pi C_m(X) \) where, since \( \Pi C_m \) is axiomless, \( X \) is nonempty. Then for each \( i, \; pr_i(x) \in C_i(pr_i(X)) \).

Since each \( C_i \) is finitary and axiomless, then there is some nonempty finite \( F_i \subset pr_i(X) \) such that \( pr_i(x) \in C_i(F_i) \subset C_i(pr_i(X)) \). Hence, nonempty and finite \( F = F_1 \times \cdots \times F_m \subset pr_1(X) \times \cdots \times pr_m(X) \). Then for each \( i, \; pr_i(F) = F_i \) implies that finite \( F = F_1 \times \cdots \times F_m \) subsequence of \( pr_1(X) \times \cdots \times pr_m(X) \). From axiom (2), \( x \in \Pi C_m(F) = C_1(pr_1(F)) \times \cdots \times C_m(pr_m(F)) \subset \Pi C_m(pr_1(X) \times \cdots \times pr_m(X)) = C_1(pr_1(X)) \times \cdots \times C_m(pr_m(X)) = \Pi C_m(X) \). This completes the proof.

References


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