ABSTRACT. A theorem for expansion of a class of functions into an integral involving associated Legendre functions is obtained in this paper. This is a somewhat general integral expansion formula for a function $f(x)$ defined in $(x_1, x_2)$ where $-1 < x_1 < x_2 < 1$, which is perhaps useful in solving certain boundary value problems of mathematical physics and of elasticity involving conical boundaries.

KEY WORDS AND PHRASES. Integral expansion of a function, associated Legendre function, Mehler-Fok integral transform.

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1. INTRODUCTION.

Integral transforms are often used to solve the problems of mathematical physics involving linear partial differential equations and also other problems. Integral expansions involving spherical functions of a class of functions are known as Mehler-Fok type transforms. In these transform formulae, the subscript of the Legendre functions appear as the integration variable while its superscript is either zero or a fixed integer (see Sneddon [10]). There is another class of integral transforms involving associated Legendre functions somewhat related to the Mehler-Fok transforms, in which the superscript of the associated Legendre function appears in the integration formula while the subscript (complex) is kept fixed. Felsen [2] first developed this type of transform formulae involving $P^{-\mu}_{-1/2+ir}(\cos \theta)$ as kernel where $0 < \theta < \pi$ from a unique $\delta$-function representation. Later Mandal ([6], [7]) obtained somewhat similar types of two transform formulae from the solution of two appropriately designed boundary value problems. In the first type, the argument $z$ of $P^{-\mu}_{-1/2+ir}(z)$ ranges from $-1$ to $1$ while in the second, the argument $z$ of $P^\mu_{-1/2+ir}(z)$ ranges from $1$ to $\infty$. Recently Mandal and Guha Roy [8] used a similar technique to establish another Mehler-Fok type integral transform formula involving $P^{-\mu}_{-1/2+ir}(\cos \theta)$ as kernel ($0 < \theta < \alpha$).

In the present paper, an integral expansion of a class of functions defined in $(x_1, x_2)$ where $-1 < x_1 < x_2 < 1$, involving associated Legendre functions is obtained. Based on direct investigation of the properties of spherical functions, sufficient conditions which would establish the validity of this expansion formula for a wide class of functions are obtained in a manner...
similar to the ideas used in ([3]-[5]). The main result is given in section 2 in the form of a theorem. Recently, we have used a similar technique to establish another type of integral representation [9] involving $P_{-1/2+ir}(\cosh \alpha)$ as kernel where $0 < \alpha < \alpha_0$.

2. INTEGRAL EXPANSION OF A FUNCTION IN $(x_1, x_2)$ WHERE $-1 < x_1 < x_2 < 1$.

We present the main result of this paper in the form of the following theorem.

**THEOREM.** Let $f(z)$ be a given function defined on the interval $(x_1, x_2)$ where $-1 < x_1 < x_2 < 1$ and satisfies the following conditions:

1. The function $f(z)$ is piecewise continuous and has a bounded variation in the open interval $(x_1, x_2)$.
2. The function $y(z)(1-z^2)\log(1-z^2)$ lies in $L(x_1, x_2)$.

Then we have

$$f(z) = \sum_{k} \sigma_k \left[ \frac{1}{2+i \tau - i \sigma_k} \right] \left[ \frac{1}{2-i \tau - i \sigma_k} \right] \frac{M(x, z; 1/2+i \sigma_k)}{M(x, z_1; 1/2+i \sigma_k)} F(\sigma_k)$$

$$+ \frac{1}{2\pi i} \int_{-\infty}^{\infty} \sigma \left[ \frac{1}{2+i \tau - i \sigma} \right] \left[ \frac{1}{2-i \tau - i \sigma} \right] \frac{M(x, z; 1/2+i \sigma)}{M(x, z_1; 1/2+i \sigma)} F(\sigma) d\sigma$$

where

$$F(\sigma) = \int_{x_1}^{x_2} \frac{f(z)}{1-z^2} M(x, z; 1/2+i \sigma) dz,$$  

$$-1 < x_1 < x_2 < 1, M(x, y; i \sigma) = p^{i \sigma}_1 - 1/2 + i \pi(x) \pi^{i \sigma}_1 - 1/2 + i \pi(-y) - \pi^{i \sigma}_1 - 1/2 + i \pi(-y)$$

and $\sigma, \epsilon, \sigma, \tau$ are real. The equation (2.2) may be regarded as an integral transform of the function $f(z)$ defined in $(x_1, x_2)$ and (2.1) is its inverse. (2.1) and (2.2) together give the integral expansion of the function $f(z)$.

**PROOF OF THE EXPANSION THEOREM.** To prove this expansion theorem, we first note that the representation (cf. Erdélyi [1])

$$p^{i \sigma}_1 - 1/2 + i \pi(x) = \left( \frac{1 + x}{1 - x} \right)^{i \sigma/2} f \left( \frac{1}{2} + i \tau, \frac{1}{2} - i \tau; 1 - i \sigma, \frac{1 - x}{2} \right)$$

$$-1 < x_1 < x < x_2 < 1,$$

where $F(a, b; c; z)$ denotes the hypergeometric series, implies $p^{i \sigma}_1 - 1/2 + i \pi(x)$ is continuous in the region defined by $-1 < x_1 < x < x_2 < 1$, $-\infty < \sigma < \infty$ and satisfies the inequality

$$\left| p^{i \sigma}_1 - 1/2 + i \pi(x) \right| \leq \sqrt{\sin \sigma} \sin \sigma P - 1/2 + i \pi(x),$$

where the Legendre function $P - 1/2 + i \pi(x)$ is positive.

Using (2.3) it follows from (2.2) that

$$\int_{x_1}^{x_2} \left( \frac{f(z)}{1-z^2} \right) \left( \frac{1}{2+i \tau - i \sigma} \right) \left( \frac{1}{2-i \tau - i \sigma} \right) \frac{M(x, z; 1/2+i \sigma)}{M(x, z_1; 1/2+i \sigma)} \left( \frac{1}{2+i \tau - i \sigma} \right) \left( \frac{1}{2-i \tau - i \sigma} \right) \frac{M(x, z; 1/2+i \sigma)}{M(x, z_1; 1/2+i \sigma)} dz$$

$$\leq \sqrt{\sin \sigma} \sin \sigma \int_{x_1}^{x_2} \left( \frac{f(z)}{1-z^2} \right) \left( \frac{1}{2+i \tau - i \sigma} \right) \left( \frac{1}{2-i \tau - i \sigma} \right) \frac{M(x, z; 1/2+i \sigma)}{M(x, z_1; 1/2+i \sigma)} \left( \frac{1}{2+i \tau - i \sigma} \right) \left( \frac{1}{2-i \tau - i \sigma} \right) \frac{M(x, z; 1/2+i \sigma)}{M(x, z_1; 1/2+i \sigma)} dz,$$

and this shows that the conditions imposed on $f(z)$ imply that the integral $F(\sigma)$ is absolutely and uniformly convergent for $\sigma \in [-T, T]$ where $T$ is a positive large number. Hence $F(\sigma)$ is continuous on $[-T, T]$ and the repeated integral

$$J(z, T) = \frac{1}{2\pi i} \int_{-T}^{T} \sigma \left( \frac{1}{2+i \tau - i \sigma} \right) \left( \frac{1}{2-i \tau - i \sigma} \right) \frac{M(x, z; 1/2+i \sigma)}{M(x, z_1; 1/2+i \sigma)} dz \int_{x_1}^{x_2} \frac{f(y)}{1-y^2} M(y, z_1; 1/2+i \sigma) dy$$
is meaningful. Also, uniform convergence allows us to change the order of integration and write $J(z,T)$ as

$$J(z,T) = \int_{x_1}^{x_2} \frac{f(y)}{1-y^2} K(z,y,T) \, dy.$$  \hfill (2.4)

where

$$K(z,y,T) = \frac{1}{2\pi i} \oint_{T} \sigma \left[ \left( \frac{1}{2} + i\sigma - i\sigma \right) \left( -i\sigma - \frac{1}{2} \right) \frac{M(z,x_2;1;\sigma) M(y,x_1;1;\sigma)}{M(x_2,x_1;1;\sigma)} \right] \, d\sigma.$$  \hfill (2.5)

Now we shall show that the kernel $K(z,y,T)$ is symmetric in the variables $x$ and $y$. By definition, we have

$$K(z,y,T) - K(y,z,T) = \frac{1}{2\pi i} \oint_{T} \sigma \left[ \left( \frac{1}{2} + i\sigma - i\sigma \right) \left( -i\sigma - \frac{1}{2} \right) \frac{M(z,x_2;1;\sigma) M(y,x_1;1;\sigma)}{M(x_2,x_1;1;\sigma)} \right] \, d\sigma.$$  \hfill (2.6)

It follows from the properties of associated Legendre functions (cf. Erdélyi [1]) that the integrand in the above integral is an odd function of $\sigma$, hence the integral vanishes. Thus

$$K(z,y,T) = K(y,z,T).$$  \hfill (2.7)

To investigate the behavior of $K(z,y,T)$ as $T \to \infty$, by writing $\mu = -i\sigma$, we write (2.5) as

$$K(z,y,T) = \frac{1}{2\pi i} \oint_{T} \mu \left[ \left( \frac{1}{2} + i\mu + \mu \right) \left( -i\mu - \frac{1}{2} \right) \frac{M(z,x_2;1;\mu) M(y,x_1;1;\mu)}{M(x_2,x_1;1;\mu)} \right] \, d\mu.$$  \hfill (2.8)

Expression under the integral sign in (2.7) is analytic function fo the complex variable $\mu$ and it has no singularity in the semi-plane $Re\mu \geq 0$, except for simple poles at $\mu = -i\sigma_k$ ($k$ is positive integer) (cf. Felsen [2]), where

$$M(x_2,x_1;1;\sigma_k) = 0, \sigma_k > 0.$$  \hfill (2.9)

Completing the contour of integration on (2.7) with the arc $\gamma_T$ of radius $T$ situated in the semi-plane $Re\mu \geq 0$ and applying the residue theorem, we obtain

$$K(z,y,T) = K_1(z,y,T) - \sum_k \sigma_k \left[ \left( \frac{1}{2} + i\sigma_k - i\sigma_k \right) \left( -i\sigma_k - \frac{1}{2} \right) \frac{M(z,x_2;1;\sigma_k) M(y,x_1;1;\sigma_k)}{M(x_2,x_1;1;\sigma_k)} \right] \, d\mu.$$  \hfill (2.10)

Suppose that $y \leq x$. By virtue of the definition

$$P^{-\mu}_{-1/2 + i\sigma} (z) = \left( \frac{1+z}{1-z} \right)^{-\mu/2} \frac{1}{(1+\mu)} \left[ 1 + 0(1 | \mu | - 1) \right],$$

$$P^{-\mu}_{-1/2 + i\sigma} (-z) = \left( \frac{1-z}{1+z} \right)^{-\mu/2} \frac{1}{(1+\mu)} \left[ 1 + 0(1 | \mu | - 1) \right].$$  \hfill (2.11)

Using (2.11) and asymptotic properties of the gamma function for large $\mu$, we conclude that

$$\mu \left[ \left( \frac{1}{2} + i\mu + \mu \right) \left( -i\mu - \frac{1}{2} \right) \frac{M(z,x_2;1;\mu) M(y,x_1;1;\mu)}{M(x_2,x_1;1;\mu)} \right]$$
Now introduce the new variables

\[
\frac{1}{2} \ln \frac{1 + x}{1 - x}, \quad \eta = \frac{1}{2} \ln \frac{1 + y}{1 - y}, \quad \alpha = \frac{1}{2} \ln \frac{1 + x_1}{1 - x_1} \quad \text{and} \quad \beta = \frac{1}{2} \ln \frac{1 + x_2}{1 - x_2}.
\]

Then, for large \( \mu \), from (2.10) - (2.12) we obtain for \( y < x \)

\[
K_1(x, y, T) = \frac{1}{2\pi i} \int \frac{\exp \{- \mu(\xi - \eta)\} + \exp \{- \mu(2\beta - 2\alpha - \xi + \eta)\}}{\pi/2} + O(1)
\]

\[
\frac{1}{2} \int_0^{\pi} \exp\{- \mu(\xi - \eta)\cos \varphi\} + \exp\{- \mu(2\beta - 2\alpha - \xi + \eta)\cos \varphi\} \, d\varphi,
\]

for \( \alpha < \eta \leq \xi < \beta \).

Using the identity

\[
\frac{1}{2} \int_0^{\pi} \exp\{- \lambda T \cos \varphi\} \, d\varphi \leq \frac{1 - \exp\{- \lambda T\}}{\lambda T}, \quad \lambda \geq 0,
\]

we obtain for \( y < x \),

\[
K_1(x, y, T) = \frac{1}{2} \left[ \frac{\sin T(\xi - \eta)}{\xi - \eta} + \frac{\sin T(2\beta - 2\alpha - \xi + \eta)}{2\beta - 2\alpha - \xi + \eta} - \frac{\sin T(\xi + \eta - 2a)}{\xi + \eta - 2a} \right]
\]

\[
- \frac{\sin T(2\beta - \xi - \eta)}{2\beta - \xi - \eta} + O(1) \left[ \frac{1 - \exp\{- T(\xi - \eta)\}}{T(\xi - \eta)} + \frac{1 - \exp\{- T(2\beta - 2\alpha - \xi + \eta)\}}{T(2\beta - 2\alpha - \xi + \eta)} \right]
\]

\[
- \frac{1 - \exp\{- T(\xi + \eta - 2a)\}}{T(\xi + \eta - 2a)} - \frac{1 - \exp\{- T(2\beta - \xi - \eta)\}}{T(2\beta - \xi - \eta)} \right]\alpha < \eta \leq \xi < \beta,
\]

where the factor \( O(1) \) is independent of \( y \).

Again for \( y \geq x \), we use the symmetry property (2.6) and the representation (2.10) of \( K_1(x, y, T) \) with the variables \( x, y \) replaced by \( y, x \).

Now we write (2.4) as

\[
J(x, T) = \frac{x}{x_1} \frac{f(y)}{1 - y^2} K_1(x, y, T) \, dy + \frac{x_2}{x_1} \frac{f(y)}{1 - y^2} K_1(x, y, T) \, dy
\]

\[
- \sum_k \sigma_k \left( \frac{1}{2} + i\tau - i\sigma_k \right) \left( \frac{1}{2} - i\tau - i\sigma_k \right) \frac{M(x, z_2; i\sigma_k)}{(\partial/\partial \sigma_k)M(x_2, z_1; i\sigma_k)} \int_{z_1}^{x_2} \frac{f(y)}{1 - y^2} M(y, z_1; i\sigma_k) \, dy
\]

\[
= J_1(x, T) + J_2(x, T) - \sum_k \sigma_k \left( \frac{1}{2} + i\tau - i\sigma_k \right) \left( \frac{1}{2} - i\tau - i\sigma_k \right) \frac{M(x, z_2; i\sigma_k)}{(\partial/\partial \sigma_k)M(x_2, z_1; i\sigma_k)} \times
\]

\[
x \int_{z_1}^{x_2} \frac{f(y)}{1 - y^2} M(y, z_1; i\sigma_k) \, dy.
\]

(2.14)
Using (2.13) in $J_1$, we obtain

\begin{align*}
J_1(z, T) &= \frac{1}{\beta} \left[ \int_{\alpha}^{\epsilon} f(\tanh \eta) \sin \frac{T(\xi - \eta)}{\xi - \eta} d\eta + \int_{\alpha}^{\epsilon} f(\tanh \eta) \sin \frac{T(2\beta - 2\alpha - \xi + \eta)}{2\beta - 2\alpha - \xi + \eta} d\eta \right. \\
& \quad - \int_{\alpha}^{\epsilon} f(\tanh \eta) \frac{\sin \frac{T(\xi + \eta - 2\alpha)}{\xi + \eta - 2\alpha}}{\xi + \eta - 2\alpha} d\eta - \int_{\alpha}^{\epsilon} f(\tanh \eta) \frac{\sin \frac{T(2\beta - \xi - \eta)}{2\beta - \xi - \eta}}{2\beta - \xi - \eta} d\eta \\
& \quad + O(1) \left[ \int_{\alpha}^{\epsilon} |f(\tanh \eta)| \frac{1 - \exp\left(-T(\xi - \eta)\right)}{T(\xi - \eta)} d\eta \right. \\
& \quad + \int_{\alpha}^{\epsilon} |f(\tanh \eta)| \frac{1 - \exp\left(-T(2\beta - 2\alpha - \xi + \eta)\right)}{T(2\beta - 2\alpha - \xi + \eta)} d\eta \\
& \quad - \int_{\alpha}^{\epsilon} |f(\tanh \eta)| \frac{1 - \exp\left(-T(\xi + \eta - 2\alpha)\right)}{T(\xi + \eta - 2\alpha)} d\eta \\
& \quad - \int_{\alpha}^{\epsilon} |f(\tanh \eta)| \frac{1 - \exp\left(-T(2\beta - \xi - \eta)\right)}{T(2\beta - \xi - \eta)} d\eta \right] \quad (2.15)
\end{align*}

The conditions satisfied by $f(z)$ imply that $f(\tanh \eta) \in L(\alpha, \beta)$; hence, by virtue of Dirichlet's theorem, for $T \to \infty$

\begin{align*}
\frac{1}{\beta} \int_{\alpha}^{\epsilon} f(\tanh \eta) \sin \frac{T(\xi - \eta)}{\xi - \eta} d\eta &= \frac{1}{\beta} f(\tanh \xi - \alpha) + o(1) \\
&= \frac{1}{2} f(z - \alpha) + o(1), \\
\frac{1}{\beta} \int_{\alpha}^{\epsilon} f(\tanh \eta) \sin \frac{T(2\beta - 2\alpha - \xi + \eta)}{2\beta - 2\alpha - \xi + \eta} d\eta &= o(1), \\
\frac{1}{\beta} \int_{\alpha}^{\epsilon} f(\tanh \eta) \sin \frac{T(\xi + \eta - 2\alpha)}{\xi + \eta - 2\alpha} d\eta &= o(1),
\end{align*}

and

\begin{align*}
\frac{1}{\beta} \int_{\alpha}^{\epsilon} f(\tanh \eta) \sin \frac{T(2\beta - \xi - \eta)}{2\beta - \xi - \eta} d\eta &= o(1).
\end{align*}

Moreover, if the integral of integration is divided into the subintervals $(\xi - \delta, \xi)$ and $(\alpha, \xi - \delta)$ and if a sufficiently small positive $\delta$ (implying a sufficiently large $T$) is chosen, then we have

\begin{align*}
\int_{\alpha}^{\epsilon} |f(\tanh \eta)| \frac{1 - \exp\left(-T(\xi - \eta)\right)}{T(\xi - \eta)} d\eta \\
&\leq \frac{1}{\beta T} \int_{\alpha}^{\epsilon} |f(\tanh \eta)| d\eta + \int_{\xi - \delta}^{\epsilon} |f(\tanh \eta)| d\eta \\
&= O(T^{-1}) + o(1) = o(1) \text{ for } T \to \infty,
\end{align*}

\begin{align*}
\int_{\alpha}^{\epsilon} |f(\tanh \eta)| \frac{1 - \exp\left(-T(2\beta - 2\alpha - \xi + \eta)\right)}{T(2\beta - 2\alpha - \xi + \eta)} d\eta \\
&\leq \frac{1}{\beta T} \int_{\alpha}^{\epsilon} |f(\tanh \eta)| d\eta \leq \frac{1}{\beta T} \int_{\alpha}^{\epsilon} |f(\tanh \eta)| d\eta.
\end{align*}
Thus (2.15) to (2.17) leads to
\[
\lim_{T \to \infty} J_1 (\tanh \xi, T) = \frac{1}{2} f(\tanh \xi - \sigma) = \frac{1}{2} f(z - \sigma).
\]
Similarly,
\[
\lim_{T \to \infty} J_2 (\tanh \xi, T) = \frac{1}{2} f(\tanh \xi + \sigma) = \frac{1}{2} f(z + \sigma).
\]
Hence,
\[
\lim_{T \to \infty} J(x, T) = \frac{1}{2} [f(z + \sigma) + f(z - \sigma)] - \sum_k \sigma_k \left( \frac{1}{2} + i\tau - i\sigma_k \right) \left( \frac{1}{2} - i\tau - i\sigma_k \right) M(x, z; \sigma_k) \left( \frac{\partial}{\partial \sigma_k} \right) M(x, z; \sigma_k) F(\sigma_k).
\]
Thus, at the points of continuity of \(f(z)\) we obtain (2.1). We note that (2.1) becomes a result in [5] when \(z_1 = -1\) and \(z_2 = 1\).

It follows from the foregoing theorem that, at points of continuity of \(f(z)\), we have
\[
f(z) = \sum_k \sigma_k \left( \frac{1}{2} + i\tau - i\sigma_k \right) \left( \frac{1}{2} - i\tau - i\sigma_k \right) \frac{R(x, z; \sigma_k)}{\partial \sigma_k} R(z, z; \sigma_k) F(\sigma_k)
\]
\[
+ \sum_{\sigma} 2i \int_{-\infty}^{\infty} \sigma \left( \frac{1}{2} + i\tau - i\sigma \right) \left( \frac{1}{2} - i\tau - i\sigma \right) \frac{R(x, z; \sigma)}{\partial \sigma} R(z, z; \sigma) F(\sigma) d\sigma,
\]
where
\[
F(\sigma) = \int_{-1}^{x_2} \frac{f(z)}{1 - x^2} R(x, z_1; i\sigma) dx, -1 < x_1 < x_2 < 1.
\]
\[
R(x, y; i\sigma) = p^{\sigma}_{-1/2 + i\tau(x)} \frac{p^{\sigma}_{-1/2 + i\tau(y)} p^{\sigma}_{-1/2 + i\tau(-y)} - p^{\sigma}_{-1/2 + i\tau(-y)} p^{\sigma}_{-1/2 + i\tau(y)}}{\partial y} d\sigma,
\]
and \(\sigma_k, \sigma, \tau\) are real.

The integrand in (2.21) has singularities at \(\sigma = \sigma_k (k\ is\ positive\ integers)\) which are simple poles along the positive \(\sigma\)-axis, where
\[
\frac{\partial}{\partial x_2} R(x, z_1; i\sigma_k) = 0, (\sigma_k > 0).
\]

To prove (2.21) we use the following asymptotic formulas for large \(\mu\):
\[
\frac{\partial}{\partial x} P^{-\mu/2 + i\tau(x)} = -\mu \left( 1 + \mu \right)^{-1/2} \left( 1 + x \right)^{-\mu/2} \left( 1 + x \right)^{-\mu/2} [1 + O(|\mu|^{-1})],
\]
\[
\frac{\partial}{\partial x} P^{-\mu/2 + i\tau(-x)} = -\mu \left( 1 + \mu \right)^{-1/2} \left( 1 - x \right)^{-\mu/2} \left( 1 + x \right)^{-\mu/2} [1 + O(|\mu|^{-1})],
\]
\[
\frac{\partial}{\partial y} P^{-\mu/2 + i\tau(y)} = -\mu \left( 1 + \mu \right)^{-1/2} \left( 1 + \mu \right)^{-\mu/2} \left( 1 + \mu \right)^{-\mu/2} [1 + O(|\mu|^{-1})].
\]
The proof of (2.21) is similar to the proof in the section 2, and we do not reproduce it. We note that (2.21) becomes a result in [5] when $x_1 = -1$ and $x_2 = 1$.

3. EXAMPLES.

We now give examples of expansions of some functions.

(1) \[ (1 - z^2)^{\nu/2} = \sum_k \sigma_k \left[ \frac{1}{2} + i\tau - i\sigma_k \right] \left[ \frac{1}{2} - i\tau - i\sigma_k \right] \frac{M(z, x_2; i\sigma_k)}{(\partial/\partial \sigma_k) M(x_2, x_1; i\sigma_k)} \]

\[ = \frac{2\nu}{(\nu^2 + \sigma_k^2)} (\nu + i\sigma_k) \left[ P_{\nu}^{-\nu}(x_1)M_1(x_1, x_2; i\sigma_k) - P_{\nu}^{-\nu}(x_2) M_1(x_2, x_1; i\sigma_k) \right] \]

\[ + \frac{2\nu}{2\pi i} \int_{-\infty}^{\infty} \frac{\sigma (\nu + i\sigma)}{\nu^2 + \sigma^2} \left[ \frac{1}{2} + i\tau - i\sigma \right] \left[ \frac{1}{2} - i\tau - i\sigma \right] M(x, x_2; i\sigma) \]

\[ - \frac{1}{M(x_2, x_1; i\sigma)} \left[ P_{\nu}^{-\nu}(x_1)M_1(x_1, x_2; i\sigma) - P_{\nu}^{-\nu}(x_2) M_1(x_2, x_1; i\sigma) \right] d\sigma, \]

where

\[ M(x, y; i\sigma) = P_{\nu}^{\nu}(x) P_{\nu}^{\nu}(y) - P_{\nu}^{\nu}(-x) P_{\nu}^{\nu}(-y), \]

\[ M_1(x, y; i\sigma) = P_{\nu}^{\nu}(-1) P_{\nu}^{\nu}(x) - P_{\nu}^{\nu}(-1) P_{\nu}^{\nu}(y) \quad \text{and} \quad \nu = -1/2 + i\tau. \]

(2) \[ P_\nu^\nu(x) = \sum_k \sigma_k \left[ \frac{1}{2} + i\tau - i\sigma_k \right] \left[ \frac{1}{2} - i\tau - i\sigma_k \right] \frac{M(x, x_2; i\sigma_k)}{(\partial/\partial \sigma_k) M(x_2, x_1; i\sigma_k)} \]

\[ \cdot \left[ (\nu + \mu) P_{\nu}^{\nu}(-1) M_1(x_2, x_1; i\sigma_k) + (\nu + i\sigma_k) \left( P_{\nu}^{\nu}(x_1)M_1(x_1, x_2; i\sigma_k) \right) \right] \]

\[ - \left[ P_{\nu}^{\nu}(x_2) M_1(x_2, x_1; i\sigma_k) \right] + \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{\sigma}{\nu^2 + \sigma^2} \left[ \frac{1}{2} + i\tau - i\sigma \right] \left[ \frac{1}{2} - i\tau - i\sigma \right] M(x, x_2; i\sigma) \]

\[ \cdot \left[ (\nu + \mu) P_{\nu}^{\nu}(-1) M_1(x_2, x_1; i\sigma) + (\nu + i\sigma) \left( P_{\nu}^{\nu}(x_1) \right) \right] \]

\[ M_1(x_1, x_2; i\sigma) - P_{\nu}^{\nu}(x_2) M_1(x_2, x_1; i\sigma) \right] d\sigma. \]

In all these results the conditions under which the expansion theorem hold are satisfied.

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