ABSTRACT. A Tychonoff non-normal space is constructed which can be used for the construction of a regular space on which every weakly continuous (hence every \( \theta \)-continuous or \( \eta \)-continuous) map into a given space is constant.

KEY WORDS AND PHRASES. Tychonoff, non-normal, weakly, \( \theta \), \( \eta \)-continuous maps.

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1. INTRODUCTION.

We construct for every Hausdorff space \( R \) a Tychonoff non-normal space \( S \) such that if \( f \) is a weakly continuous map of \( S \) into \( R \) then there exist two closed subsets \( K', L', K' \cap L' = \emptyset \) such that \( f(K') = f(L') = \{ r \} \), \( r \in R \). Therefore, applying the method of Jones [1], we can first construct a regular space containing two points \(-\infty, +\infty\) such that \( f(-\infty) = f(+\infty) \), for every weakly continuous map \( f \) of this space into \( R \) and then, applying the method of Illiadis and Tzannes [2], a regular space on which every weakly continuous (hence every \( \theta \)-continuous or \( \eta \)-continuous (Dickman, Porter and Rubin [3])) map into \( R \) is constant. The construction of \( S \) is a modification of the space \( T(R) \) in Illiadis and Tzannes [2]. For regular spaces on which every continuous map into a given space is constant see also Armentrout [4], Brandenburg and Mysior [5], van Douwen [6], Herrlich [7], Hewitt [8], Tzannes [9] and Jounglove [10]. A map \( f : X \to Y \), where \( X, Y \) are topological spaces is called
1) weakly continuous if for every \( x \in X \) and open neighbourhood of \( f(x) \) there exists an open neighbourhood \( V \) of \( x \), such that \( f(V) \subseteq \text{Cl}U \),
2) \( \theta \)-continuous if for every \( x \in X \) and open neighbourhood \( U \) of \( f(x) \), there is an open neighbourhood \( V \) of \( x \) such that \( f(\text{Cl}V) \subseteq \text{Cl}U \)
3) \( \eta \)-continuous if for every regular-open sets \( U, V \) of \( Y \),
\[ (i) \quad f^{-1}(U) \subseteq \text{IntCl}^{-1}(U) \]
\[ (ii) \quad \text{IntCl}^{-1}(U \cap V) \subseteq \text{IntCl}^{-1}(U) \cap \text{IntCl}^{-1}(V). \]

Every \( \eta \)-continuous is \( \theta \)-continuous (Dickman, Porter and Rubin [3, Proposition 3.3. (c)]) and every \( \theta \)-continuous is obviously weakly continuous.

We denote
1) by \( |X| \) the cardinality, of \( X \),
2) by \( \psi(X) = \sup \{ \psi(X, x) : x \in X \} \) the pseudocardinal of \( X \), where \( \psi(X, x) \) is the pseudocardinal of \( X \) at \( x \), that is the minimal cardinality of pseudobases of \( x \). (The set \( U_\alpha \) consisting of open neighbourhoods of \( x \), is called a pseudobasis if \( \cap U_\alpha = \{ x \} \),
3) by \( \psi^+(X) \) the smallest cardinal number greater than \( \psi(X) \).

2. THE SPACE \( S \).

Let \( R \) be a Hausdorff space and \( K, L \) two uncountable sets such that \( |K| = |L| = \mathbb{R} > |R| \).
For every $k_i \in K$ (resp. $l_i \in L$) we consider an uncountable set $K_i$ (resp. $L_i$) and a set $M$ such that $|K_i| = |L_i| = |M| \geq \psi^+(R)$. On the set $S = M \cup KU \cup K_i \cup LU \cup L_i$ we define the following topology: Every point belonging to $K_i, L_i$ is isolated. For every $k_i \in K$ (resp. $l_i \in L$) a basis of open neighbourhoods are the sets $O(k_i) = \{k_i\} \cup C_i$ (resp. $O(l_i) = \{l_i\} \cup D_i$), where $C_i, D_i$ consist of all but finite number of elements of $K_i, L_i$ respectively. For every point $m \in M$ a basis of open neighbourhoods are the sets $O(m) = \{m\} \cup P \cup Q$, where $P, Q$ contain all but finite number of elements of the sets $\{h_i(m) : i \in I\}, \{g_i(m) : i \in I\}$, respectively, where $I$ is an index set, $|I| = \aleph_0$ and $h_i, g_i$ are one-to-one maps of $M$ onto $K_i, L_i$, respectively.

One can show that the space $S$ is Tychonoff and non-normal.

Let $f$ be a weakly continuous map of $S$ into $R$. Since $|K| > |R|$, it follows that for some $r_1 \in R$ there exists $K' \subseteq K$ such that $|K'| = |K|$ and $f(K') = \{r_1\}$. Let $\{k_n : n = 1, 2, \ldots\}$ be a countable subset of $K'$. Since for every open neighbourhood $U$ of $r_1$ the set $f^{-1}(CIU)$ contains an open neighbourhood of $k_n, n = 1, 2, \ldots$, it follows that $|K_n \setminus f^{-1}(r_1)| \leq \psi(R, r_1)$. Consequently, if $h_n$ is the one-to-one map of $M$ onto $K_n$ then $h_n^{-1}(K_n \setminus f^{-1}(r_1)) \leq \psi(R, r_1)$ and hence $\bigcup_{n=1}^{\infty} h_n^{-1}(K_n \setminus f^{-1}(r_1)) \leq \psi(R, r_1)$. Repeating all the above for the set $L$ we have that for some $r_2 \in R$ there exist $L' \subseteq L, |L'| = |L|, f(L') = \{r_2\}$ and a countable subset $\{l_n : n = 1, 2, \ldots\} \subseteq L'$ such that if $V$ is an open neighbourhood of $r_2$ then $|L_n \setminus f^{-1}(r_2)| \leq \psi(R, r_2)$ and hence $\bigcup_{n=1}^{\infty} g_n^{-1}(L_n \setminus f^{-1}(r_2)) \leq \psi(R, r_2)$. Therefore if $M' = \bigcup_{n=1}^{\infty} (h_n^{-1}(K_n \setminus f^{-1}(r_1)) \cup g_n^{-1}(L_n \setminus f^{-1}(r_2)))$ then $M \setminus M' \neq \emptyset$. Let $m \in M \setminus M'$ and $CIW$ be a closed neighbourhood of $f(m)$ such that $r_1, r_2 \not\in CIW$. There exists an open neighbourhood $O(m)$ of $m$ such that $f(O(m)) \subseteq CIW$, while for every $n = 1, 2, \ldots, h_n(m) \in f^{-1}(r_1), g_n(m) \in f^{-1}(r_2)$ which imply that $f(m) = r_1 = r_2$.

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