Letter to the Editor

Periodic Solution of the Hematopoiesis Equation

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Wu and Liu (2012) presented some results for the existence and uniqueness of the periodic solutions for the hematopoiesis model. This paper gives a simple approach to find an approximate period of the model.

Wu and Liu studied the following hematopoiesis model [1]:

\[ x' (t) = -ax(t) + \frac{\beta \theta^n}{\theta^n + x^n (t-\tau)}, \]  

where \( x \) denotes the density of mature cells in blood circulation. The physical meaning of other parameters is referred to [1].

Equation (1) admits periodic solutions as revealed in [1]. Hereby we suggest a simple approach to the search for an approximate period of (1) using a simple amplitude-frequency formulation [2–5]. To this end, we rewrite (1) in the form

\[ x'(t) \theta^n + x'(t) x^n (t-\tau) + ax(t) \theta^n + ax(t) x^n (t-\tau) - \beta \theta^n = 0. \]  

Assume that the periodic solution can be expressed in the form

\[ x(t) = A \cos \omega t. \]  

Submitting (3) into (2) results in the following residual:

\[ R(\omega, t) = -A \omega \theta^n \sin \omega t - A^{1+n} \omega \sin \omega t \cos^n \omega (t-\tau) + a \theta^n A \cos \omega t \]  

\[ + a A^{1+n} \cos \omega t \cos^n \omega (t-\tau) - \beta \theta^n. \]  

In order to use the amplitude-frequency formulation [2–5], we choose two trial frequencies and locate them at \( t = \pi/(4\omega) \).

Setting \( \omega_1 = 1 \), \( \omega_1 t = \pi/4 \), and \( \omega_2 = 2 \), \( \omega_2 t = \pi/4 \), respectively, we have

\[ R_1 = -\frac{\sqrt{2}}{2} A \theta^n - \frac{\sqrt{2}}{2} A^{1+n} \cos^n \left( \frac{\pi}{4} - \tau \right) \]  

\[ + \frac{\sqrt{2}}{2} a \theta^n A + \frac{\sqrt{2}}{2} a A^{1+n} \cos^n \left( \frac{\pi}{4} - \tau \right) - \beta \theta^n, \]  

\[ R_2 = -\frac{\sqrt{2}}{2} A \theta^n - \frac{\sqrt{2}}{2} A^{1+n} \cos^n \left( \frac{\pi}{4} - 2\tau \right) \]  

\[ + \frac{\sqrt{2}}{2} a \theta^n A + \frac{\sqrt{2}}{2} a A^{1+n} \cos^n \left( \frac{\pi}{4} - 2\tau \right) - \beta \theta^n. \]  

The frequency can be then obtained approximately in the form [2–5]

\[ \omega^2 = \frac{R_1 \omega_1^2 - R_2 \omega_2^2}{R_1 - R_2} = \frac{R_1 - 4R_2}{R_1 - R_2} \]  

\[ = \left( \frac{7}{2} \sqrt{2} A \theta^n + \frac{7}{2} a A^{1+n} \cos^n \left( \frac{\pi}{4} - 2\tau \right) - \frac{3 \sqrt{2}}{2} a \theta^n ight) \]  

\[ - \frac{3}{2} a A^{1+n} \cos^n \left( \frac{\pi}{4} - 2\tau \right) + 3 \beta \theta^n. \]
\[ \times \left( \sqrt{2} - \frac{\sqrt{2}}{2} \right) A \theta^n \]

\[ - \left( \sqrt{2} - \frac{\sqrt{2}}{2} \right) A^{1+n} \cos^n \left( \frac{\pi}{4} - \tau \right)^{-1}. \]

(6)

This formulation has been widely used to solve periodic solutions of various nonlinear oscillators [6–13], and it is often called as He's frequency formulation, He's amplitude-frequency formulation, or He's frequency-amplitude formulation. In case \( \omega^2 < 0 \), no period solution is admitted. A similar criterion is given for a nonlinear equation arising in electrospinning process [14].

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**References**


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