On a New Class of Multivalent Functions
With Missing Coefficients

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Abstract

In this paper, we investigate a new class $Θ_{ξ₁,ξ₂}^{p,λ}$ of analytic functions in the open unit disk. By using the geometry function theory, we discuss the radius problems between the $Θ_{ξ₁,ξ₂}^{p,λ}$ and the convex functions or close-to-convex functions. Several properties as the sufficient and necessary conditions and modified-Hadamard product are given.

Keywords: Multivalent function, Convex function, Cauchy-schwarz inequality, Modified-Hadamard product.

1 Introduction

Let $A_p$ be the class of functions of the form

$$f(z) = z^p + \sum_{n=p+1}^{∞} a_n z^n, \quad p \in Z^+ = \{1, 2, 3, \ldots\},$$

that are $p$-valently analytic in the open unit disk $U = \{z \in C : |z| < 1\}$. If two functions $f_1(z) \in A_p$, $f_2(z) \in A_p$ and

$$f_i(z) = z^p + \sum_{n=p+1}^{∞} a_{n,i} z^n, \quad i = 1, 2, z \in U,$$
then we define the \( f_1 \oplus f_2(z) \) as
\[
f_1 \oplus f_2(z) = z^p + \sum_{n=p+1}^{\infty} (a_{n,1} + a_{n,2})z^n, \ z \in \mathbb{U}.
\]

Also, let \( K_p(\alpha) \) denote the subclass of \( \mathcal{A}_p \) consisting of \( f(z) \) which satisfy
\[
\Re\left(1 + \frac{zf''(z)}{f'(z)}\right) > \alpha, \quad (z \in \mathbb{U})
\]
for some real \( \alpha (0 \leq \alpha < p) \). A function \( f(z) \in K_p(\alpha) \) is said to be \( p \)-valently convex of order \( \alpha \) in \( \mathbb{U} \). We note that \( K_1(\alpha) \equiv K \) is usual convex class. Moreover, a function \( f(z) \in \mathcal{A}_p \) is in the class \( C_p(\alpha) \) if
\[
\Re\left(\frac{f'(z)}{pz^{p-1}}\right) > \alpha, \quad z \in \mathbb{U}
\]
for some real \( \alpha (0 \leq \alpha < 1) \). \( C_1(0) \equiv C \) is the close-to-convex class. These are many results on the classes \( K_p(\alpha) \) and \( C_p(\alpha) \) (See [1, 2, 8, 9, 10, 13]).

Let \( \mathcal{A}_p(\theta) \) denote the subclass of \( \mathcal{A}_p \) consisting of functions \( f(z) \) with the coefficients \( a_n = |a_n|e^{i((n-p)\theta+\pi)} \) \( (n \geq p+1) \). Here, we introduce the subclasses \( C_p(\theta, \alpha) \) and \( K_p(\theta, \alpha) \) as follows: \( C_p(\theta, \alpha) = \mathcal{A}_p(\theta) \cap C_p(\alpha) \), \( K_p(\theta, \alpha) = \mathcal{A}_p(\theta) \cap K_p(\alpha) \). In fact, The \( C_1(\theta, \alpha) \) was introduced by Uyanik, Owa [12] and the \( K_1(\theta, \alpha) \equiv K(\theta, \alpha) \) was introduced by Frasin [7].

In some earlier investigations, various interesting subclasses of the class \( \mathcal{A}_p \) and \( \mathcal{A}_p(\theta) \) have been studied with different view points (see [3, 4]). Motivated by the aforementioned works done by Uyanik et al. [11, 12] and Frasin et al. [5, 6, 7], we now introduce the following subclass \( \Theta^{p,\lambda}_{\xi_1, \xi_2} \) of analytic functions:

**Definition 1.1** For the functions \( f(z) \in \mathcal{A}_p \) given by (1), we say that \( f(z) \in \Theta^{p,\lambda}_{\xi_1, \xi_2} \), if there exists a function \( g(z) = z^p + \sum_{n=p+1}^{\infty} b_n z^n \in \mathcal{G} \) such that
\[
\left|\xi_1 z \left(\frac{f(z) \oplus g(z)}{z^p}\right) + \xi_2 z^2 \left(\frac{f(z) \oplus g(z)}{z^p}\right)''\right| \leq \lambda, \ z \in \mathbb{U},
\]
where \( \xi_1, \xi_2 \in \mathbb{C}, \ \lambda > 0, \ p \in \mathbb{Z}^+ \) and
\[
\mathcal{G} = \left\{ g(z) \in \mathcal{A}_p : b_{p+1} = 0, b_{p+2} = -\frac{1}{2}a_{p+2}, \right. \\
b_{p+3} = -\frac{2}{3}a_{p+3}, ..., b_n = \left(\frac{1}{n-p} - 1\right)a_n, ... \left. \right\}.
\]

In the present paper, some properties for \( \Theta^{p,\lambda}_{\xi_1, \xi_2} \) are given. We discuss the radius problems for \( f(z) \) belonging to \( C_p(\theta, \alpha) \) or \( K_p(\theta, \alpha) \) to be in the class \( \Theta^{p,\lambda}_{\xi_1, \xi_2} \), and obtain the modified-Hadamard product results.
2 Sufficient and Necessary Conditions

Theorem 2.1 If the function \( f(z) \) given by (1) satisfies the condition
\[
\sum_{n=p+1}^{\infty} [|\xi_1| + |\xi_2|(n-p-1)]|a_n| \leq \lambda, \tag{6}
\]
then \( f(z) \in \Theta_{\xi_1, \xi_2}^{p, \lambda} \) with a function
\[
g(z) = z^p + \sum_{n=p+1}^{\infty} b_n z^n \in \mathcal{G},
\]
where \( \xi_1, \xi_2 \in \mathbb{C}, \lambda > 0 \) and \( p \in \mathbb{Z}^+ = \{1, 2, 3, ...\} \).

Proof For \( f(z) \in \mathcal{A}_p \) and \( g(z) \in \mathcal{G} \), using the (5), then we have
\[
\left| \xi_1 z \left( \frac{f(z) \oplus g(z)}{z^p} \right) \right| + \xi_2 z^2 \left( \frac{f(z) \oplus g(z)}{z^p} \right)''
\]
\[
= \left| \sum_{n=p+1}^{\infty} [\xi_1 (n-p) + \xi_2 (n-p)(n-p-1)](a_n + b_n)z^{n-p} \right|
\]
\[
\leq \sum_{n=p+1}^{\infty} [|\xi_1| (n-p) + |\xi_2|(n-p)(n-p-1)]|a_n + b_n|
\]
\[
= \sum_{n=p+1}^{\infty} [|\xi_1| + |\xi_2|(n-p-1)]|a_n|.
\]
It follows from (4), (6) and (7), then \( f(z) \in \Theta_{\xi_1, \xi_2}^{p, \lambda} \). The proof of the theorem is complete.

Theorem 2.2 If \( f(z) = z^p + \sum_{n=p+1}^{\infty} a_n z^n \in \Theta_{\xi_1, \xi_2}^{p, \lambda} \) with a function
\[
g(z) = z^p + \sum_{n=p+1}^{\infty} b_n z^n \in \mathcal{G},
\]
and \( \arg \xi_1 = \arg \xi_2 = \gamma \) and \( a_n = |a_n|e^{i((n-p)\theta-\gamma)} \), then we have
\[
\sum_{n=p+1}^{\infty} [|\xi_1| + |\xi_2|(n-p-1)]|a_n| \leq \lambda.
\]
Proof If \( f(z) \in \Theta_{\xi_1,\xi_2}^{p,\lambda} \) with \( \arg \xi_1 = \arg \xi_2 = \gamma \) and \( a_n = |a_n|e^{i((n-p)\theta-\gamma)} \), applying the (5), then we get

\[
\left| \xi_1 z \left( \frac{f(z) \oplus g(z)}{z^p} \right)' + \xi_2 z^2 \left( \frac{f(z) \oplus g(z)}{z^p} \right)'' \right| =
\]

\[
= \left| \sum_{n=p+1}^{\infty} [\xi_1(n-p) + \xi_2(n-p)(n-p-1)](a_n + b_n)z^{n-p} \right|
\]

\[
= \sum_{n=p+1}^{\infty} [\xi_1 + \xi_2(n-p-1)]a_n z^{n-p}
\]

\[
= \sum_{n=p+1}^{\infty} [||\xi_1| + |\xi_2|(n-p-1)||a_n|e^{i(n-p)\theta-\gamma}z^{n-p}]
\]

\[
= \sum_{n=p+1}^{\infty} [||\xi_1| + |\xi_2|(n-p-1)||a_n||z^{n-p} \leq \lambda
\]

for all \( z \in \mathbb{U} \). Letting \( z \in \mathbb{U} \) such that \( z = |z|e^{-i\theta} \), then we have that

\[
\left| \sum_{n=p+1}^{\infty} [||\xi_1| + |\xi_2|(n-p-1)||a_n|e^{i(n-p)\theta}z^{n-p} \right|
\]

\[
= \sum_{n=p+1}^{\infty} [||\xi_1| + |\xi_2|(n-p-1)||a_n||z^{n-p} \leq \lambda
\]

Now, taking \( |z| \to 1^- \), form (8) and (9), it gives the required result. The proof of the theorem is complete.

3 Radius Problems with Convex and Close-to-Convex Functions

Working in a similar way as in Uyanik, Owa [11, Lemma 3.1] and Frasin [6, Lemma 4.1], we give the following Lemma 3.1 and Lemma 3.2:

Lemma 3.1 Suppose \( f(z) = z^p + \sum_{n=p+1}^{\infty} a_n z^n \in C_p(\theta, \alpha) \), then we have

\[
\sum_{n=p+1}^{\infty} n|a_n| \leq p(1 - \alpha), (0 \leq \alpha < 1).
\]
Lemma 3.2 Suppose \( f(z) = z^p + \sum_{n=p+1}^{\infty} a_n z^n \in \mathcal{K}_p(\theta, \alpha) \), then we have
\[
\sum_{n=p+1}^{\infty} \frac{n}{p} (n - \alpha)|a_n| \leq p - \alpha, \quad (0 \leq \alpha < p).
\]

Theorem 3.3 Let \( f(z) = z^p + \sum_{n=p+1}^{\infty} a_n z^n \in \mathcal{C}_p(\theta, \alpha) \) and \( \delta(0 < |\delta| < 1) \) is a complex number, then \( \frac{1}{\delta^p} f(\delta z) \in \Theta^{p,\lambda}_{\xi_1,\xi_2} \) with a function \( g(z) \in G \) for \( 0 < |\delta| \leq |\delta_0(\lambda)| \), where \( |\delta_0(\lambda)| \) is the smallest positive root of the equation
\[
|\xi_1||\delta|\sqrt{p(1 - \alpha)(1 - |\delta|^2)} + |\xi_2|\sqrt{1 + |\delta|^2}|\delta|^2 \sqrt{p(1 - \alpha) - |a_{p+1}|^2 - \lambda(1 - |\delta|^2)^2} = 0.
\]

Proof If \( f(z) \in \mathcal{C}_p(\theta, \alpha) \), then we have that
\[
\frac{1}{\delta^p} f(\delta z) = z^p + \sum_{n=p+1}^{\infty} a_n \delta^{n-p} z^n.
\]

Applying Theorem 2.1, we need to show that
\[
\sum_{n=p+1}^{\infty} [||\xi_1| + |\xi_2|(n - p - 1)||a_n||\delta|^{n-p} \leq \lambda.
\]

By using the Cauchy–Schwarz inequality, we can obtain
\[
\sum_{n=p+1}^{\infty} [||\xi_1| + |\xi_2|(n - p - 1)||a_n||\delta|^{n-p} \leq \lambda.
\]

In fact, Lemma 3.1 implies that
\[
\sum_{n=p+1}^{\infty} |a_n|^2 \leq \sum_{n=p+1}^{\infty} |a_n| \quad \leq \sum_{n=p+1}^{\infty} n|a_n| \leq p(1 - \alpha),
\]
So we also have
\[ \sum_{n=p+1}^{\infty} |a_n|^2 \leq p(1 - \alpha) - |a_{n+1}|^2. \] (12)

Moreover, putting \( x = |\delta|^2 \), then we have
\[ \sum_{n=p+1}^{\infty} |\delta|^{2n} = \sum_{n=p+1}^{\infty} x^n = \frac{x^{p+1}}{1 - x} \] (13)

and
\[ \sum_{n=p+2}^{\infty} (n - p - 1)^2 |\delta|^{2n} \]
\[ = \sum_{n=p+2}^{\infty} (n - p - 1)^2 x^n = \frac{1 + x}{(1 - x)^3} x^{p+2}. \] (14)

Following (10)-(14), we can obtain that
\[ \sum_{n=p+1}^{\infty} [|\xi_1| + |\xi_2|(n - p - 1)]|a_n||\delta|^{n-p} \]
\[ \leq \frac{|\xi_1|}{|\delta|^p} \left( \sum_{n=p+1}^{\infty} |\delta|^{2n} \right)^{\frac{1}{2}} \left( \sum_{n=p+1}^{\infty} |a_n|^2 \right)^{\frac{1}{2}} \]
\[ + \frac{|\xi_2|}{|\delta|^p} \left( \sum_{n=p+2}^{\infty} (n - p - 1)^2 |\delta|^{2n} \right)^{\frac{1}{2}} \left( \sum_{n=p+2}^{\infty} |a_n|^2 \right)^{\frac{1}{2}} \]
\[ \leq \frac{|\xi_1|}{|\delta|^p} \left( \frac{x^{p+1}}{1 - x} \right)^{\frac{1}{2}} \left( p(1 - \alpha) \right)^{\frac{1}{2}} \]
\[ + \frac{|\xi_2|}{|\delta|^p} \left( \frac{1 + x}{(1 - x)^3} x^{p+2} \right)^{\frac{1}{2}} \left( p(1 - \alpha) - |a_{p+1}|^2 \right)^{\frac{1}{2}} \]
\[ \leq \frac{|\xi_1|}{|\delta|^p} \left( \frac{x^{p+1}}{1 - x} \right)^{\frac{1}{2}} \left( p(1 - \alpha) \right)^{\frac{1}{2}} \]
\[ + \frac{|\xi_2|}{|\delta|^p} \left( \frac{1 + x}{(1 - x)^3} x^{p+2} \right)^{\frac{1}{2}} \left( p(1 - \alpha) - |a_{p+1}|^2 \right)^{\frac{1}{2}} \]
\[ = |\xi_1| \frac{\sqrt{p(1 - \alpha)}}{(1 - |\delta|^2)^{\frac{1}{2}}} + |\xi_2| \frac{\sqrt{1 + |\delta|^2} |\delta|^2 \sqrt{p(1 - \alpha) - |a_{p+1}|^2}}{(1 - |\delta|^2)^{\frac{3}{2}}} \].
We need to consider the complex number \( \delta(0 < |\delta| < 1) \) such that
\[
|\xi_1|\sqrt{p(1-\alpha)} + |\xi_2|\sqrt{1+|\delta|^2}\sqrt{p(1-\alpha) - |a_{p+1}|^2} = \lambda.
\]

Hence, we definite the following function with \(|\delta(\lambda)|\) by
\[
F(|\delta(\lambda)|) = |\xi_1||\delta|\sqrt{p(1-\alpha)}(1-|\delta|^2)
+ |\xi_2|\sqrt{1+|\delta|^2}\sqrt{p(1-\alpha) - |a_{p+1}|^2} - \lambda(1-|\delta|^2)^2/2.
\]

It is easily to know that \( F(0) = -\lambda < 0 \) and
\[
F(1) = \sqrt{2}|\xi_2|\sqrt{p(1-\alpha) - |a_{p+1}|^2} > 0,
\]
which implies that there exists some \( \delta_0(\lambda) \) such that \( F(|\delta_0(\lambda)|) = 0(0 < |\delta_0(\lambda)| < 1) \). The proof of the theorem is complete.

**Theorem 3.4** Let \( f(z) = z^p + \sum_{n=p+1}^{\infty} a_n z^n \in \mathcal{K}_p(\theta, \alpha) \) and \( \delta(0 < |\delta| < 1) \) is a complex number. Then \( \frac{1}{\delta^p} f(\delta z) \in \Theta_{|\xi_1|,|\xi_2|}^{p,\lambda} \) with a function \( g(z) \in \mathcal{G} \) for \( 0 < |\delta| \leq |\delta_0(\lambda)| \), where \(|\delta_0(\lambda)|\) is the smallest positive root of the equation
\[
|\xi_1||\delta|\sqrt{p-\alpha(1-|\delta|^2)} + |\xi_2|\sqrt{1+|\delta|^2}\sqrt{p-\alpha - |a_{p+1}|^2 - \lambda(1-|\delta|^2)^2} = 0.
\]

**Proof** Since \( f(z) \in \mathcal{K}_p(\theta, \alpha) \), using Lemma 3.2, we have that
\[
\sum_{n=p+1}^{\infty} \frac{n}{p}(n-\alpha)|a_n| \leq p - \alpha,
\]
which leads to
\[
\sum_{n=p+1}^{\infty} |a_n|^2 \leq \sum_{n=p+1}^{\infty} (n-p)|a_n|^2 \leq \sum_{n=p+1}^{\infty} \frac{n}{p}(n-\alpha)|a_n|^2 \leq \sum_{n=p+1}^{\infty} \frac{n}{p}(n-\alpha)|a_n| \leq p - \alpha.
\]
Hence, from (15), we can also note that
\[
\sum_{n=p+1}^{\infty} \left[ |\xi_1| + |\xi_2|(n - p - 1)\right]|a_n||\delta|^{n-p} \leq |\xi_1| \left( \sum_{n=p+1}^{\infty} |a_n|^2 \right)^{1/2} \left( \sum_{n=p+1}^{\infty} |\delta|^{2n} \right)^{1/2} + |\xi_2| \left( \sum_{n=p+2}^{\infty} (n - p - 1)^2|\delta|^{2n} \right)^{1/2} \left( \sum_{n=p+2}^{\infty} |a_n|^2 \right)^{1/2}\]
\[
+ \frac{|\xi_2|}{|\delta|^{p}} \left( \frac{1 + x}{(1 - x)^3} x^{p+2} \right)^{1/2} \left( p - \alpha - |a_{p+1}|^2 \right)^{1/2} + |\xi_2| \frac{\sqrt{1 + |\delta|^2} \sqrt{|\delta|^2} \sqrt{p - \alpha - |a_{p+1}|^2}}{(1 - |\delta|^2)^{3/2}} \]
\]

Using the same technique as in the proof of Theorem 3.3, we derive the result. The proof of the theorem is complete.

4 Modified-Hadamard Product

Let \( f(z) = z^p + \sum_{n=p+1}^{\infty} |a_n|e^{i((n-p)\theta)-\gamma}z^n \), \( g(z) = z^p + \sum_{n=p+1}^{\infty} |b_n|e^{i((n-p)\theta)-\gamma}z^n \).

We define modified Hadamard product for the functions \( f, g \) as follows:
\[
(f \ast g)(z) = z^p + \sum_{n=p+1}^{\infty} |a_n||b_n|e^{i((n-p)\theta)-\gamma}z^n, \quad z \in \mathbb{U}.
\]

**Theorem 4.1** If \( f_1(z) = z^p + \sum_{n=p+1}^{\infty} |a_{n,1}|e^{i((n-p)\theta)-\gamma}z^n \in \Theta_{\xi_1,\xi_2}^{p,\lambda_1} \) with \( g_1(z) \in \mathcal{G} \), \( f_2(z) = z^p + \sum_{n=p+1}^{\infty} |a_{n,2}|e^{i((n-p)\theta)-\gamma}z^n \in \Theta_{\xi_1,\xi_2}^{p,\lambda_2} \) with a function \( g_2(z) \in \mathcal{G} \) and \( \arg \xi_1 = \arg \xi_2 = \gamma \), then we have
\[
(f_1 \ast f_2)(z) = z^p + \sum_{n=p+1}^{\infty} |a_{n,1}||a_{n,2}|e^{i((n-p)\theta)-\gamma}z^n \in \Theta_{\xi_1,\xi_2}^{p,\lambda^*}
\]
with a function \( g(z) \in \mathcal{G} \), where
\[
\lambda^* = \frac{1}{|\xi_1| \lambda_1 \lambda_2}.
\]
Proof Suppose \( f_1(z) = z^p + \sum_{n=p+1}^{\infty} |a_{n,1}|e^{i((n-p)\theta-\gamma)}z^n \in \Theta^{p,\lambda_1}_{\xi_1,\xi_2} \), \( f_2(z) = z^p + \sum_{n=p+1}^{\infty} |a_{n,2}|e^{i((n-p)\theta-\gamma)}z^n \in \Theta^{p,\lambda_2}_{\xi_1,\xi_2} \) and \( \arg \xi_1 = \arg \xi_2 = \gamma \), then from Theorem 2.2, we have
\[
\sum_{n=p+1}^{\infty} \left[ \frac{|\xi_1| + |\xi_2|(n-p-1)|a_{n,1}|}{\lambda_1} \right] \leq 1 \tag{18}
\]
and
\[
\sum_{n=p+1}^{\infty} \left[ \frac{|\xi_1| + |\xi_2|(n-p-1)|a_{n,2}|}{\lambda_2} \right] \leq 1. \tag{19}
\]
Moreover, (18) and (19) imply that
\[
\left\{ \sum_{n=p+1}^{\infty} \left[ \frac{|\xi_1| + |\xi_2|(n-p-1)|a_{n,1}|}{\lambda_1} \right] ^\frac{1}{2} \leq 1 \right\} \tag{20}
\]
and
\[
\left\{ \sum_{n=p+1}^{\infty} \left[ \frac{|\xi_1| + |\xi_2|(n-p-1)|a_{n,2}|}{\lambda_2} \right] ^\frac{1}{2} \leq 1 \right\}. \tag{21}
\]
By using the Holder inequality with (20) and (21), we get
\[
\sum_{n=p+1}^{\infty} \left\{ \frac{|\xi_1| + |\xi_2|(n-p-1)}{\lambda_1} \right\} ^\frac{1}{2} \left\{ \frac{|\xi_1| + |\xi_2|(n-p-1)}{\lambda_2} \right\} ^\frac{1}{2} \left| a_{n,1} \right| \left| a_{n,2} \right| \leq 1,
\]
so
\[
\sum_{n=p+1}^{\infty} |\xi_1| + |\xi_2|(n-p-1)\left\{ \frac{1}{\lambda_1} \right\} ^\frac{1}{2} \left\{ \frac{1}{\lambda_2} \right\} ^\frac{1}{2} \left| a_{n,1} \right| \left| b_{n,2} \right| \leq 1. \tag{22}
\]
In order to obtain the \((f \ast g)(z) \in \Theta^{p,\lambda^*}_{\xi_1,\xi_2}\) with a function \(g(z) \in \mathcal{G}\), we have to find the corresponding \(\lambda^*\) such that
\[
\sum_{n=p+1}^{\infty} \left[ \frac{|\xi_1| + |\xi_2|(n-p-1)|a_{n,1}|b_{n,2}|}{\lambda^*} \right] \leq 1. \tag{23}
\]
Following (22), then (23) hold true if for any \(n \geq p+1\),
\[
\frac{1}{\lambda^*} \leq \left( \frac{1}{\lambda_1} \right) ^\frac{1}{2} \left( \frac{1}{\lambda_2} \right) ^\frac{1}{2} \frac{1}{\sqrt{|a_{n,1}|b_{n,2}|}}
\]
or
\[
\lambda^* \geq (\lambda_1) ^\frac{1}{2} (\lambda_2) ^\frac{1}{2} \sqrt{|a_{n,1}|b_{n,2}|}. \tag{24}
\]
In fact, (24) implies that

$$\lambda^* = \max\{ \mathcal{L}(n) | \mathcal{L}(n) = (\lambda_1)^{\frac{1}{2}}(\lambda_2)^{\frac{1}{2}} \sqrt{|a_{n,1}|b_{n,1}|}, \forall n \geq 1 + p \}.$$ 

Furthermore, from (22), it is easy to know that

$$\sqrt{|a_{n,1}|b_{n,1}|} \leq \frac{1}{|\xi_1| + |\xi_2|/(n-p-1)}(\lambda_1\lambda_2)^{\frac{1}{2}},$$

(25)

since $|\xi_1| + |\xi_2|(n-p-1)$ is increasing in $n$, following (25), then we can see that

$$\mathcal{L}(n) = (\lambda_1)^{\frac{1}{2}}(\lambda_2)^{\frac{1}{2}} \sqrt{|a_{n,1}|b_{n,1}|} \leq \frac{1}{|\xi_1| + |\xi_2|/(n-p-1)} \lambda_1\lambda_2$$

$$\leq \frac{1}{|\xi_1| + |\xi_2|/(n-p-1)}_{n=p+1} \lambda_1\lambda_2 = \frac{1}{|\xi_1|} \lambda_1\lambda_2.$$

The proof of the theorem is complete.

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References


