A Study of EOQ Model with Power Demand of Deteriorating Items under the Influence of Inflation

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Abstract

The objective of this model is to investigate the inventory system for perishable items with power demand pattern where two parameter Weibull distribution for deterioration is considered. The Economic order quantity is determined for minimizing the average total cost per unit time under the influence of inflation and time value of money. Here the deterioration starts after a fixed time interval. The influences of inflation and time-value of money on the inventory system are investigated with the help of some numerical examples.

Keywords: Inventory system, Power demand, Deterioration, Weibull distribution.
1.0. Introduction

Deterioration of items in inventory systems has become an interesting feature for its practical importance. Deterioration refers to damage, spoilage, vaporization, or obsolescence of the products. There are several types of items that will deteriorate if stored for extended periods of time. Examples of deteriorating items include metal parts, which are prone to corrosion and rusting, and food items, which are subject to spoilage and decay. Electronic components and fashion clothing also fall into this category, because they can become obsolete over time and their demand will decrease drastically.

A number of researchers have worked on inventory models with time-varying demand. Goyal and Giri (2001)[4] provided the most recent review of the literature on deteriorating inventory models published since the early 1990s, classifying them on the basis of shelf-life characteristic and demand variations. For many products, the influence of various market trends on demand is a primary factor in deterioration. A few researchers have discussed the demand of items as power demand pattern. Datta and Pal[2] have developed an order level inventory system with power demand pattern, assuming the deterioration of items governed by a special form of Weibull density function \( \theta(t) = \theta_0 t; 0 < \theta_0 << 1, t > 0 \) . They used the special form of this function to sidetrack the mathematical complication in deriving a compact EOQ model.

Goswami and Chaudhuri (1992)[3] formulated and analytically solved the inventory replenishment problem for a deteriorating item with linearly time-varying demand, finite shortage cost, and equal replenishment intervals. Wee (1995)[8] proposed a replenishment policy for a deteriorating product where demand declines exponentially over a fixed time horizon with constant deterioration and complete or partial backordering. Models were numerically solved and the policies compared assuming that shortages are allowed. Su and Lin (2001) [6] optimally solved a production-inventory model for deteriorating products with shortages, in which the demand is exponentially decreasing and the production rate at any time depends on both the demand and the inventory level. They determined optimal expressions for the production period, maximum inventory level, backorder level, and average total cost.

Shorthages may or may not be allowed; and partially or completely back-ordered; or lost. Padmanabhan and Vrat (1995)[5] developed stock-dependent demand models and determined economic order quantity (EOQ) expressions for deteriorating items with complete, partial, and no backlogging. Wee (1995)[8] and Chu and Chen (2002)[1] allowed shortages for all periods except the final one, assuming a fixed ratio of backorders to lost sales. Chu and Chen (2002)[1] showed that the inventory carrying cost is proportional to the cost of deteriorated items, and proposed a near-optimal closed-form approximate solution. Wang (2002) [7] argued that assuming a fixed fraction of backorders is not reasonable, since the length of the waiting period is the main factor in whether or not customers accept backordering. He introduced a time dependent partial backlogging rate and opportunity cost due to lost sales.
In this paper an economic order quantity model is developed for perishable items with power demand pattern using two-parameter Weibull density function for deterioration. In shortage state during stock out it is assumed that all demands are backlogged or lost. The backlogging rate is variable and dependent on the waiting time for the next replenishment. We have considered the time-value of money and inflation of each cost parameter. Life-time of the items is also taken into consideration.

2.0. Assumptions and Notations

Following assumptions are made for the proposal model:

i. Single inventory will be used.

ii. Lead time is zero.

iii. The model is studied when shortages are allowed.

iv. Demand follows the power demand pattern, i.e., demand up to time $t$ is assumed to be $d \left( \frac{t}{T} \right)^{1/n}$, where $d$ is the demand size during the planning horizon $T$, $n$ $(0 < n < ∞)$ is the pattern index.

v. When shortages are allowed, it is also partially backlogged. The backlogging rate is variable and depends on the length of waiting time for the next replacement. The backlogging rate is assumed to be $\frac{1}{1 + δ(T - t)}$ where $δ$ is the non negative constant backlogging parameter.

vi. Replenishment rate is infinite but size is finite.

vii. Time horizon is finite.

viii. There is no repair of deteriorated items occurring during the cycle.

ix. Deterioration occurs when the item is effectively in stock.

x. The time-value of money and inflation are considered.

xi. The second and higher powers of $α$ and $δ$ are neglected in this analysis of the model hereafter.

Following notations are made for the given model:

$I(t) = \text{On hand inventory level at any time } t, t ≥ 0$.

$R(t) = \left[ \frac{d t^{(1-n)/n}}{nT^{1/n}} \right]$ is the demand rate at time $t$.

$θ : θ = αβ t^{β-1}$, The two-parameter Weibull distribution deterioration rate (unit/unit time). Where $0 < α << 1$ is called the scale parameter, $β > 0$ is the shape parameter.
\( Q \) = Total amount of replenishment in the beginning of each cycle.

\( V \) = Inventory at time \( t = 0 \)

\( T \) = Duration of a cycle.

\( \sigma \) = The life-time of items.

\( i \) = The inflation rate per unit time.

\( r \) = The discount rate representing the time value of money.

\( c_p \) = The purchasing cost per unit item.

\( c_d \) = The deterioration cost per unit item.

\( c_h \) = The holding cost per unit item.

\( c_o \) = The opportunity cost due to lost sales per unit.

\( c_b \) = The shortage cost per unit.

\( U \) = The total average cost of the system.

**3.0. Formulation**

The objective of the model is to determine the optimal order quantity in order to keep the total relevant cost as low as possible. The optimality is determined for shortage of items. Taking \( Q \) be the total amount of replenishment in the beginning of each cycle, and after fulfilling backorders let \( V \) be the level of initial inventory. In the period \((0, \sigma)\) the inventory level gradually decreases due to market demand only.

During the period \((\sigma, t_f)\) the inventory stock further decreases due to combined effect of deterioration and demand. At \( t_f \), the level of inventory reaches zero and after that the shortages are allowed to occur during the interval \([t_f, T]\). Here part of shortages is backlogged and part of it is lost sale. Only the backlogging items are replaced by the next replenishment. The behavior of inventory during the period \((0, T)\) is depicted in the following inventory-time diagram.
Here we have taken the total duration $T$ as fixed constant. The objective here is to determine the optimal order quantity in order to keep the total relevant cost as low as possible.

If $I(t)$ be the on-hand inventory at time $t \geq 0$, then at time $t + \Delta t$, the on-hand inventory in the interval $[0, \sigma]$ will be

$$I(t + \Delta t) = I(t) - R(t) \Delta t$$

Dividing by $\Delta t$ and then taking as $\Delta t \to 0$ we get

$$\frac{dI}{dt} = -\frac{dt^{(1-\alpha)/n}}{nT^{1/n}}; \quad 0 \leq t \leq \sigma$$

For the next interval $[\sigma, t_1]$, where the effect of deterioration starts with the presence of demand, i.e.,

$$I(t + \Delta t) = I(t) - \theta(t) I(t) \Delta t - R(t) \Delta t$$

Dividing by $\Delta t$ and then taking as $\Delta t \to 0$ we get

$$\frac{dI(t)}{dt} + \alpha \beta t^{\beta-1} I(t) = -\frac{dt^{(1-\alpha)/n}}{nT^{1/n}}; \quad \sigma \leq t \leq t_1$$

Finally in the interval $[t_1, T]$, where the shortages are allowed

$$I(t + \Delta t) = I(t) - \frac{R(t)}{1 + \delta(T-t)} \cdot \Delta t$$

Dividing by $\Delta t$ and then taking as $\Delta t \to 0$ we get

$$\frac{dI(t)}{dt} = -\frac{dt^{(1-\alpha)/n}}{1 + \delta(T-t)}; \quad t_1 \leq t \leq T$$

The boundary conditions are $I(0) = V$ and $I(t_1) = 0$.

On solving equation (3.1) with boundary condition we have

$$I(t) = V - \frac{dt^{1/n}}{T^{1/n}}; \quad 0 \leq t \leq \sigma$$

On solving equation (3.2) with boundary condition we have

$$I(t) = \frac{d}{T^{1/n}} \left[ (1 - \alpha t^{\beta}) (t^{1/n}_1 - t^{1/n}) + \frac{\alpha}{\beta + 1} (t^{1/n}_1 - t^{1/n}) \right]; \quad \sigma \leq t \leq t_1$$
On solving equation (3.3) with boundary condition we have

\[ I(t) = \frac{d}{T^{1/n}} \left[ (1 - \delta t)(t_1^{1/n} - t^{1/n}) + \frac{\delta}{n+1} \left( t_1^{1/n} - t^{1/n} \right) \right] ; \quad t_1 \leq t \leq T \]

Now the initial inventory is given by,

\[ V = \frac{d}{T^{1/n}} e^{-\alpha \sigma} \left[ (t_1^{1/n} - \sigma^{1/n}) + \frac{\alpha}{n\beta + 1} \left( t_1^{1/n} - \sigma^{1/n} \right) \right] + \frac{d \sigma^{1/n}}{T^{1/n}} \]

The total cost function consists of the following elements if the inflation and time-value of money are considered:

(i) **Purchasing cost per cycle**

\[ C_p V \int_0^T e^{-(r-i)t} dt = \frac{C_p V}{i-r} \left[ e^{-(r-i)T} - 1 \right] \]

(ii) **Holding cost per cycle**

\[ C_h \int_0^T \left( I(t). e^{-(r-i)t} + C_h \right) \left( I(t). e^{-(r-i)t} \right) dt \]

\[ = C_h \left[ V \sigma + \frac{V(i-r)}{2} - \frac{d n}{(n+1)} T^{1/n} \sigma n - \frac{d n(i-r)}{(2n+1)} T^{1/n} \sigma n \right] \]

\[ + \frac{C_h d}{T^{1/n}} \left[ t_1^{1/n} \left\{ \{1 - \sigma\} - \frac{\alpha n}{\beta + 1} \left\{ \beta^{1/n} - \sigma^{1/n} \right\} \right\} - \frac{\alpha n}{n\beta + n+1} \left\{ \beta^{1/n} - \sigma^{1/n} \right\} \right] \]

\[ + \frac{\alpha n}{n\beta + 1} \int_0^{1/n} \left\{ t_1^{1/n} - \sigma^{1/n} \right\} \left[ \frac{\alpha n}{\beta + 1} \left\{ \beta^{1/n} - \sigma^{1/n} \right\} \right] \]

\[ - \frac{n(i-r)}{2n+1} \left\{ t_1^{1/n} - \sigma^{1/n} \right\} + \frac{n\alpha(i-r)}{2n + \beta + 1} \left\{ t_1^{1/n} - \sigma^{1/n} \right\} \]

\[ - \frac{n\alpha(i-r)}{(2n + \beta + 1)(n\beta + 1)} \left\{ t_1^{1/n} - \sigma^{1/n} \right\} \]
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### (iii) Deterioration cost per cycle

(3.9) \[ C_d \int^{t_i} \alpha \beta t^{\beta-1} I(t) e^{-\gamma(t-i)} dt \]

\[
= \frac{C_d \alpha \beta d}{T^{1/n}} \left[ t_i^{1/n} \int^{t_i} \left( t_i^\beta - \sigma^\beta \right) - \frac{n}{n \beta + 1} \left\{ \frac{2 n \beta + 1}{t_i^{1/n} - \sigma^n} \right\} \right] - \frac{\alpha \beta}{2 \beta} \int^{t_i} \left( t_i^\beta - \sigma^\beta \right) dt + \frac{\alpha n}{2 n \beta + 1} \left\{ \frac{2 n \beta + 1}{t_i^{1/n} - \sigma^n} \right\} \\
+ \frac{\alpha \beta}{\beta (1 + n \beta)} \left\{ t_i^\beta - \sigma^\beta \right\} - \frac{\alpha n}{(2 n \beta + 1)(n \beta + 1)} \left\{ \frac{2 n \beta + 1}{t_i^{1/n} - \sigma^n} \right\} + \frac{(i - r) t_i^\beta}{(1 + \beta)} \left\{ \frac{2 n \beta + 1}{t_i^{1/n} - \sigma^n} \right\} - \frac{(i - r) n}{(n \beta + n + 1)} \left\{ \frac{2 n \beta + 1}{t_i^{1/n} - \sigma^n} \right\} \\
- \frac{\alpha(i - r) n}{(1 + 2 \beta)} \left\{ \frac{2 n \beta + 1}{t_i^{1/n} - \sigma^n} \right\} + \frac{\alpha(i - r) n}{(2 n \beta + n + 1)} \left\{ \frac{2 n \beta + 1}{t_i^{1/n} - \sigma^n} \right\} + \frac{\alpha(i - r) n}{(1 + n \beta)(1 + \beta)} \left\{ \frac{2 n \beta + 1}{t_i^{1/n} - \sigma^n} \right\} \\
- \frac{\alpha(i - r) n}{(2 n \beta + n + 1)(n \beta + 1)} \left\{ \frac{2 n \beta + 1}{t_i^{1/n} - \sigma^n} \right\}
\]

### (iv) Shortage cost per cycle

(3.10) \[ C_o \int^{T} - I(t) e^{-\gamma(t-i)} dt \]

\[
= \frac{C_o d}{T^{1/n}} \left[ \left( 1 - \delta T \right) t_i^{1/n} (T - t_i) - \frac{n (1 - \delta T) t_i^{1/n}}{n + 1} + \frac{\delta}{n + 1} t_i^{1/n} (T - t_i) - \frac{n \delta}{(n + 1)(2 n + 1)} t_i^{1/n} \right] \\
+ \frac{(1 - \delta T) (i - r)}{2} t_i^{1/n} (T^2 - t_i^2) - \frac{(1 - \delta T) (i - r) n}{2 n + 1} t_i^{1/n} (T^2 - t_i) \left( \frac{2 n + 1}{n + 1} \right) + \frac{\delta (i - r) n}{2 (n + 1)} t_i^{1/n} (T^2 - t_i) \\
- \frac{\delta (i - r) n}{(n + 1)(3 n + 1)} t_i^{1/n} (T^2 - t_i) 
\]

### (v) Opportunity cost due to lost sales per cycle

(3.11) \[ C_o \int^{T} R(t) \left[ 1 - \frac{1}{1 + \delta (T - t)} \right] e^{-\gamma(t-i)} dt \]
\[ \frac{C_d\delta}{nT^{1/n}} \left[ nT\left( T^{1/n} - t_1^{1/n} \right) - \frac{n}{n+1} \left\{ \frac{n+1}{T^n} - t_1^n \right\} \right] \]
\[ + \frac{n(i-r)T}{n+1} \left\{ \frac{n+1}{T^n} - t_1^n \right\} - \frac{n(i-r)}{2n+1} \left\{ \frac{2n+1}{T^n} - t_1^n \right\} \]

Taking the relevant costs mentioned above, the total average cost per unit time of the system is given by

(3.12) \[ U = \frac{1}{T} \{ \text{Purchasing cost} + \text{Holding cost} + \text{Deterioration cost} + \text{Shortage cost} + \text{Opportunity cost} \} \]

\[ = \frac{1}{T} \left[ \frac{C_p V}{i-r} e^{-(i-r)T} - 1 \right] \]
\[ + C_h \left[ V\sigma + \frac{V(i-r)}{2} \sigma^2 - \frac{d n}{(n+1)T^{1/n}} \right. \]
\[ \left. \frac{n+1}{\sigma^n} - \frac{d n(i-r)}{(2n+1)T^{1/n}} \frac{2n+1}{\sigma^n} \right] \]
\[ + C_d \frac{d}{T^{1/n}} \left[ t_1^{1/n} \left\{ t_1 - \sigma \right\} - \frac{\alpha_1^{1/n}}{\beta+1} \left\{ \sigma^{\beta+1} - \sigma^{\beta+2} \right\} \right] \]
\[ - \frac{n(i-r)}{2n+1} \left\{ \frac{2n+1}{t_1^n} - \sigma^{\beta+2} \right\} + \frac{n\alpha(i-r)}{2n+n\beta+1} \left\{ \frac{2n+n\beta+1}{t_1^n} - \sigma^{\beta+2} \right\} \]
\[ - \frac{n\alpha(i-r)}{(2n+n\beta+1)(n\beta+1)} \left\{ \frac{2n+n\beta+1}{t_1^n} - \sigma^{\beta+2} \right\} \]
\[ + C_{\text{d}} \frac{\alpha \beta d}{T^{1/n}} \left[ \frac{t_1^{1/n}}{\beta} \left\{ t_1^{\beta} - \sigma^{\beta} \right\} - \frac{n}{n\beta+1} \left\{ \frac{n\beta+1}{t_1^n} - \sigma^{\beta} \right\} - \frac{\alpha_1^{1/n}}{2\beta} \left\{ \sigma^{\beta} - \sigma^{2\beta} \right\} + \frac{\alpha n}{2n+1} \left\{ \frac{2n+1}{t_1^n} - \sigma^{\beta} \right\} \right] \]
\[ + \frac{\alpha_1^{1/n}}{\beta(1+n\beta)} \left\{ \sigma^{\beta} - \sigma^{\beta+1} \right\} - \frac{\alpha}{(2n+1)(n\beta+1)} \left\{ \frac{2n+1}{t_1^n} - \sigma^{\beta+1} \right\} \]
\[ - \frac{(i-r)\alpha}{n \beta (n+1)} \left\{ \frac{n\beta+1}{t_1^n} - \sigma^{\beta+1} \right\} \]
\[ + \frac{\alpha(i-r)}{(1+2\beta)(n\beta+n+1)} \left\{ \frac{2n+1}{t_1^n} - \sigma^{\beta+1} \right\} - \frac{\alpha(i-r)}{(1+n\beta)(1+\beta)} \left\{ \frac{2n+1}{t_1^n} - \sigma^{\beta+1} \right\} \]
\[
\alpha(i-r)n \left( \frac{2n\beta+n+1}{n+1} \right) \left\{ \frac{1}{n} \left( -\sigma^2 \right) \right\} \\
- \frac{C_h d}{T^{1/n}} \left[ (1-\delta\Gamma) T^{1/n} (T - t_i) - \frac{n(1-\delta\Gamma)}{n+1} \left( T^n - t_i^n \right) + \frac{\delta}{n+1} \frac{n^{n+1}}{n} \left( T^n - t_i^n \right) \right] \\
- \frac{(1-\delta\Gamma)(i-r) \frac{1}{2} T^{n} (T^2 - t_i^2)}{n+1} - \frac{(1-\delta\Gamma)(i-r) n \frac{2n+1}{n+1} (T^n - t_i^n) \frac{2n+1}{n+1} (T^n - t_i^n)}{n+1} + \frac{\delta(i-r) \frac{n^{n+1}}{n+1} (T^n - t_i^n)}{n+1} \\
+ \frac{n(i-r)T}{n+1} \left\{ \frac{1}{n} \left( T^n - t_i^n \right) \right\} - \frac{n(i-r) \frac{2n+1}{n+1} (T^n - t_i^n)}{2n+1} \left\{ \frac{2n+1}{n+1} (T^n - t_i^n) \right\} \\
\]

Now equation (3.12) can be minimized but as it is difficult to solve the problem by deriving a closed equation of the solution of equation (3.12), Matlab Software has been used to determine optimal \( t_1^* \) and hence the optimal cost \( U(t_1^*) \) can be evaluated. Also level of initial inventory level \( V^* \) can be determined.

### 4.0. Examples

**Example- 1:**

The values of the parameters are considered as follows:

\( c_h = $3/ unit/ year, c_p = $4/ unit, c_d = $8/ unit, c_b = $12/ unit c_o = $5/ unit \)
\( r = 0.02, i = 0.38, \alpha = 0.001, \beta = 1.5, \delta = 0.1, n = 2, \sigma = 0, T = 1 year, d = 60 units. \)

Now using equation (3.12) which can be minimized to determine optimal \( t_1^* = 10.8 \text{ months} \) and hence the average optimal cost \( U(t_1^*) = $317.51/ unit. \)

Also level of initial inventory level \( V^* = 56.93 \text{ units}. \)

**Example- 2:**

The values of the parameters are considered as follows:

\( c_h = $3/ unit/ year, c_p = $4/ unit, c_d = $8/ unit, c_b = $12/ unit c_o = $5/ unit \)
\( r = 0.02, i = 0.38, \alpha = 0.001, \beta = 1.5, \delta = 0.1, n = 2, \sigma = 0.2, T = 1 year, d = 60 units. \)

Now using equation (3.12) which can be minimized to determine optimal \( t_1^* = 11.88 \text{ months} \) and hence the optimal cost \( U(t_1^*) = $300.96/ unit. \)

Also level of initial inventory level \( V^* = 59.71 \text{ units}. \)
5.0. Conclusion

Here an EOQ model is derived for perishable items with power demand pattern. Two-parameter Weibull distribution for deterioration is used. The model is studied for minimization of total average cost under the influence of inflation and time-value of money. Numerical examples are used to illustrate the result.

References


