μ− Geodetic Iteration Number and
μ− Geodetic Number of a Fuzzy Graph

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Abstract

In this paper the concepts of μ−geodesic, μ−eccentricity, μ−radius,
μ−diameter, μ−center, μ−geodetic closure, μ−geodetic iteration number are
introduced. It is proved that if $G : (V, \sigma, \mu)$ is a connected fuzzy graph on $n$
nodes such that each pair of nodes is joined by a strong arc then the μ−distance
between two nodes is the reciprocal of its arc length. Also the concepts of
μ−convex set, μ−geodetic cover, μ−geodetic basis, μ−geodetic number, μ−check
node, μ−convex hull, μ−hull number are introduced. A sufficient condition for
a fuzzy graph to have its node set as μ−geodetic basis is obtained. μ−peripheral
vertex, μ−peripheral path and μ−eccentric vertex of fuzzy graph are analyzed.

Keywords: μ−geodesic, μ−eccentricity, μ−radius, μ−geodetic closure,
μ−geodetic iteration number, μ−convex set, μ−geodetic cover, μ−geodetic ba-
sis, μ−geodetic number, μ−check node, μ−convex hull, μ−hull number.

1 Introduction

Fuzzy graphs are introduced by Rosenfeld [8]. Rosenfeld has obtained the
fuzzy analogue of several graph theoretic concepts like paths, cycles, trees
and connectedness and established some of the properties [8]. Bhattacharya
has introduced fuzzy groups and metric notion in fuzzy graphs. Bhutani and
Rosenfeld have introduced the concept of strong arcs [1] and geodesic distance
in fuzzy graphs [2]. The definition of a geodesic basis, median are also given
by the same author. Several important works on fuzzy graphs can be found in [9]. Some metric aspects using the \( \mu \)-distance is defined by Rosenfeld [8] and further studied by Sunitha and Vijayakumar [11]. In this paper, geodetic number and geodetic number of fuzzy graphs based on \( \mu \)-distance is introduced.

## 2 Preliminaries

The following definitions are from [8], [1], [7], [6] and [10].

A fuzzy graph is denoted by \( G : (V, \sigma, \mu) \) where \( V \) is a vertex set, \( \sigma \) is a fuzzy subset of \( V \) and \( \mu \) is a fuzzy relation on \( \sigma \), i.e., \( \mu(x, y) \leq \sigma(x) \land \sigma(y) \) \( \forall x, y \in V \). We consider fuzzy graph \( G \) with no loops and assume that \( V \) is finite and nonempty, \( \mu \) is reflexive (i.e., \( \mu(x, x) = \sigma(x), \forall x \)) and symmetric(i.e., \( \mu(x, y) = \mu(y, x), \forall (x, y) \)). In all the examples \( \sigma \) is chosen suitably. Also, we denote the underlying crisp graph by \( G^* : (\sigma^*, \mu^*) \) where \( \sigma^* = \{ u \in V : \sigma(u) > 0 \} \) and \( \mu^* = \{ (u, v) \in V \times V : \mu(u, v) > 0 \} \). The fuzzy graph \( H : (\tau, \nu) \) is said to be a partial fuzzy subgraph of \( G : (\sigma, \mu) \) if \( \nu \subseteq \mu \) and \( \tau \subseteq \sigma \). Let \( P \subseteq V \), the fuzzy graph \( H : (P, \tau, \nu) \) is called a fuzzy subgraph of \( G : (V, \sigma, \mu) \) induced by \( P \) if \( \tau(x) = \sigma(x) \forall x \in P \) and \( \nu(x, y) = \mu(x, y) \forall x, y \in P \). \( G : (V, \sigma, \mu) \) is called trivial if \(|\sigma^*| = 1\).

A path \( P \) of length \( n \) is a sequence of distinct nodes \( u_0, u_1, ..., u_n \) such that \( \mu(u_{i-1}, u_i) > 0, \forall i = 1, 2, ..., n \) and the degree of membership of a weakest arc is defined as its strength. If \( u_0 = u_n \) and \( n \geq 3 \) then \( P \) is called a cycle and \( P \) is called a fuzzy cycle, if it contains more than one weakest arc. The strength of a cycle is the strength of the weakest arc in it. The strength of connectedness between two nodes \( x \) and \( y \) is defined as the maximum of the strength of all paths between \( x \) and \( y \) and is denoted by \( CONN_G(x, y) \). A fuzzy graph \( G : (\sigma, \mu) \) is connected if for every \( x, y \) in \( \sigma^* \), \( CONN_G(x, y) > 0 \). A fuzzy graph \( G \) is said to be complete if \( \mu(u, v) = \sigma(u) \land \sigma(v), \forall u, v \in \sigma^* \). A connected fuzzy graph \( G : (\sigma, \mu) \) is a fuzzy tree if it has a fuzzy spanning subgraph \( F : (\sigma, \nu) \), which is a tree where for all arcs \( (x, y) \) not in \( F \) there exists a path from \( x \) to \( y \) in \( F \) whose strength is more than \( \mu(x, y) \). An arc of a fuzzy graph is called strong if its weight is at least as great as the connectedness of its end nodes when it is deleted. Depending on \( CONN_G(x, y) \) of an arc \( (x, y) \) in a fuzzy graph \( G \), Sunil Mathew and M.S.Sunitha [10] defined three different types of arcs. Note that \( CONN_{G^*-(x,y)}(x, y) \) is the the strength of connectedness between \( x \) and \( y \) in the fuzzy graph obtained from \( G \) by deleting the arc \( (x, y) \). An arc \( (x, y) \) in \( G \) is \( \alpha- \) strong if \( \mu(x, y) > CONN_{G^*-(x,y)}(x, y) \). An arc \( (x, y) \) in \( G \) is \( \beta- \) strong if \( \mu(x, y) = CONN_{G^*-(x,y)}(x, y) \). An arc \( (x, y) \) in \( G \) is \( \delta- \) arc if \( \mu(x, y) < CONN_{G^*-(x,y)}(x, y) \). A fuzzy cut node \( w \) is a node in \( G \) whose removal reduces the strength of connectedness between some pair of nodes in
$G$. If $\mu(u, v) > 0$, then $u$ and $v$ are called neighbors. Also $v$ is called strong neighbor of $u$ if arc $(u,v)$ is strong. A node $z$ is a fuzzy end node of $G$ if it has exactly one strong neighbor in $G$.

For any path $P: u_0, u_1, ..., u_n$ the $\mu$-length of $P$, $l(P)$ is defined as the sum of reciprocals of arc weights. That is $l(P) = \sum^{n}_{i=1} \frac{1}{\mu(u_{i-1}, u_i)}$. If $n = 0$ define $l(P) = 0$, and $\mu$-distance $d_\mu(u, v)$ is the smallest $\mu$-length of any $u-v$ path.

3 $\mu$- Geodesics in Fuzzy Graph

In crisp graph the concept of geodesic and geodesic iteration number are discussed in [3] and [4]. Here we are extending these ideas to fuzzy graphs. Depending on $\mu$-distance we define $\mu$-geodesic, $\mu$-eccentricity, $\mu$-radius, $\mu$-diameter, $\mu$-center, $\mu$-geodetic closure and $\mu$-geodetic iteration number as follows.

**Definition 3.1** Any path $P$ from $x$ to $y$ with smallest $\mu$-length is called $\mu$-geodesic from $x$ to $y$. i.e., Any path $P$ from $x$ to $y$ whose $\mu$-length is $d_\mu(u, v)$ is called $\mu$-geodesic from $x$ to $y$.

**Definition 3.2** The $\mu$-eccentricity $e_\mu(u)$ of a node $u$ in $G$ is given by

$$ e_\mu(u) = \text{Max}_{v \in V} d_\mu(u, v) $$

The minimum $\mu$-eccentricity among the vertices of $G$ is its $\mu$-radius denoted by $r_\mu(G)$.

$$ r_\mu(G) = \text{Min}_{v \in V} e_\mu(u) $$

A node $v$ is a $\mu$-central node if,

$$ e_\mu(v) = r_\mu(G) $$

Let $C_\mu(G)$ be the set of all $\mu$-central nodes of $G$. Then the fuzzy subgraph induced by $C_\mu(G)$ denoted by $< C_\mu(G) >$ is called $\mu$-center of $G$.

The maximum $\mu$-eccentricity among the vertices of $G$ is its $\mu$-diameter denoted by $d_\mu(G)$.

$$ d_\mu(G) = \text{Max}_{v \in V} e_\mu(u) $$

A node $v$ is a $\mu$-peripheral node or $\mu$-diametral node if,
Example 3.3 Consider the fuzzy graph given in Fig.1.

Here $\mu-$ peripheral nodes are $u$ and $y$.
$\mu-$ central nodes are $x$ and $v$.

$r_\mu(G) = 2.68$.
$d_\mu(G) = 4.21$.

Definition 3.4 Let $S$ be a set of nodes of a connected fuzzy graph $G : (V, \sigma, \mu)$. Then the $\mu-$ geodetic closure of $S$ is the set of all nodes that lie on $\mu-$ geodesics between nodes of $S$ denoted by $(S_\mu)$.

Example 3.5 Consider the fuzzy graph given in Fig.1.

If $S = \{u, w\}$.
Then $(S_\mu) = \{u, v, w\}$.
Similarly if $S = \{u, x, y\}$.
Then $(S_\mu) = \{u, v, w, x, y\}$.

4 $\mu-$ Geodetic Iteration Number for a Fuzzy Graph $[\mu-\text{gin}(G)]$

Let $S$ be a set of nodes of a connected fuzzy graph $G : (V, \sigma, \mu)$. Let $S_{\mu}^1, S_{\mu}^2, ...$, are $\mu-$ closures where $S_{\mu}^1 = (S_\mu), S_{\mu}^2 = (S_{\mu}^1) = ((S_\mu))$ etc. Since we consider only finite fuzzy graphs, the process of taking closures must terminate with some smallest $n$ such that $S_{\mu}^n = S_{\mu}^{n-1}$. That is repeat the closure operation until the stability occurs.

Definition 4.1 The smallest value of $n$ so that $S_{\mu}^n = S_{\mu}^{n-1}$ is called $\mu-$ geodetic iteration number of $S$ denoted by $\mu-\text{gin}(S)$. Now $\mu-\text{gin}(G)$ is the maximum value of $\mu-\text{gin}(S)$, for all $S \subset V(G)$. 
Remark 4.2 For a trivial fuzzy graph $G$, $\mu - \text{gin}(G) = 0$.

Example 4.3 Consider the fuzzy graph given in Fig.1.

Taking $S = \{u, x, y\}$
$S_1^1 = (S_\mu) = \{u, x, v, w, y\}$
$S_\mu^2 = S_1^1$
Therefore
$\mu - \text{gin}(S) = 2$.
It can be verified that maximum value of $\mu - \text{gin}(S) = 2$ for all $S \subset V(G)$.
Therefore
$\mu - \text{gin}(G) = 2$.

Theorem 4.4 Let $G : (V, \sigma, \mu)$ be a connected fuzzy graph on $n$ nodes such that each pair of nodes is joined by a strong arc. Then

$$d_\mu(u, v) = \frac{1}{\mu(u,v)}.$$ 

Also

$$d_\mu(u, v) = \frac{1}{\text{CONN}_G(u,v)}.$$ 

Proof
Given that all arcs in $G$ are strong. Thus $G$ contain only $\alpha-$ strong and $\beta-$ strong arcs. Therefore we have two cases.

Case.1
Let $(u, v)$ be an arc in $G$ which is $\beta-$ strong. Consider all other $u - v$ paths in $G$. Then the weight of the weakest arc in any $u - v$ path is $\mu(u, v)$. Therefore
$\text{CONN}_G(u, v) = \mu(u, v)$. (By definition of $\beta-$ strong)

Now let $P : u = u_0, u_1, ..., u_n = v$ be such a $u - v$ path. Then the $\mu-$ length of the path $P$ is

$$l(P) = \sum_{i=1}^{n} \frac{1}{\mu(u_{i-1}, u_i)} > \frac{1}{\mu(u, v)}$$

Also $\mu-$distance $d_\mu(u, v)$ is the smallest $\mu-$length of any $u - v$ path. Therefore

$$d_\mu(u, v) = \frac{1}{\mu(u,v)}.$$ 

Case.2
Let $(u, v)$ be an arc in $G$ which is $\alpha-$ strong. Then
$\text{CONN}_G(u, v) = \mu(u, v)$. (By definition of $\alpha-$ strong).
Consider all other $u-v$ paths in $P$. Let $P : u = u_0, u_1, ..., u_n = v$ be such a $u - v$ path and $(x, y)$ be an arc in G. Then
\[ \mu(x, y) < \mu(u, v) \quad \text{(By definition of} \alpha-\text{strong)} \]

i.e.,

\[ \frac{1}{\mu(x, y)} > \frac{1}{\mu(u, v)} \]

Hence

\[ l(P) = \sum_{i=1}^{n} \frac{1}{\mu(u_{i-1}, u_i)} > \frac{1}{\mu(u, v)} \]

Also \( \mu- \) distance \( d_\mu(u, v) \) is the smallest \( \mu- \) length of any \( u-v \) path. Therefore

\[ d_\mu(u, v) = \frac{1}{\mu(u, v)} \]

If the arc is \( \alpha- \) strong or \( \beta- \) strong, then

\[ \mu(u, v) = \text{CONN}_{G}(u, v) \ [10] \]

Therefore

\[ d_\mu(u, v) = \frac{1}{\text{CONN}_{G}(u, v)}. \]

Hence the proof.

**Corollary 4.5** For a complete fuzzy graph \( G : (V, \sigma, \mu) \) on \( n \) nodes

\[ d_\mu(u, v) = \frac{1}{\mu(u, v)}. \]

**Remark 4.6** For a complete fuzzy graph \( G \), each arc is a \( \mu- \) geodesic between its end nodes. So when we consider any \( S \subseteq V(G) \), any pair of nodes in \( S \) is connected by a \( \mu- \) geodesic, i.e., no \( \mu- \) geodesic between a pair of nodes of \( S \) contains another node. So \( S_\mu^1 = (S_\mu) = S \). This is true for any \( S \subseteq V(G). \) Hence \( \mu- \) gin\( (G) = 1 \) for a complete fuzzy graph \( G \).

**Remark 4.7** The converse of Theorem 4.4 need not be true. That is if \( G : (V, \sigma, \mu) \) is a connected fuzzy graph with \( d_\mu(u, v) = \frac{1}{\mu(u, v)} \) for each arc \( (u, v) \)

\( \forall u, v \in V(G) \), it does not imply that each pair of nodes in \( G \) is joined by a strong arc.

**Example 4.8** Consider the fuzzy graph given in Fig.2.
Here for each arc \((u,v)\) we have \(d_\mu(u,v) = \frac{1}{\mu(u,v)}\). But arc \((u,v)\) and arc \((u,w)\) are not strong arcs.

5 \(\mu\)– Geodetic Number of a Fuzzy Graph \([\mu–gn(G)]\)

Depending on \(\mu\)–distance we define \(\mu\)–convex set, \(\mu\)–geodetic cover, \(\mu\)–geodetic basis, and \(\mu\)–geodetic number of a fuzzy graph as follows. Then a sufficient condition for a fuzzy graph to have its node set as \(\mu\)–geodetic basis is obtained.

**Definition 5.1** A set \(S\) is \(\mu\)–convex if all nodes on any \(\mu\)–geodesic between two of its nodes are contained in \(S\). Thus \(S\) is convex if \((S_\mu) = S\).

**Example 5.2** Consider the fuzzy graph given in Fig.1.

If \(S = \{u, v, w\}\), then \((S_\mu) = S\). Therefore \(S\) is a \(\mu\)–convex set.

**Definition 5.3** A \(\mu\)–geodetic cover of \(G\) is a set \(S \subseteq V(G)\) such that every node of \(G\) is contained in a \(\mu\)–geodesic joining some pair of nodes in \(S\).

**Example 5.4** Consider the fuzzy graph given in Fig.1.
If \(S = \{u, x, y\}\).
Then \((S_\mu) = \{u, w, x, v, y\} = V(G)\).
Therefore \(S\) is a \(\mu\)–geodetic cover.

Consider the fuzzy graph given in Fig.2.
If \(S = \{u, v, x, w\}\).
Then \((S_\mu) = \{u, v, x, w\} = V(G)\).
Therefore \(S\) is a \(\mu\)–geodetic cover.

**Proposition 5.5** A connected fuzzy graph has at least one \(\mu\)–geodetic cover.

**Definition 5.6** The \(\mu\)–geodetic number of \(G\) denoted by \(\mu–gn(G)\), is the minimum order of its \(\mu\)–geodetic covers and any cover of order \(\mu–gn(G)\) is a \(\mu\)–geodetic basis.
**Example 5.7** Consider the fuzzy graph given in Fig.1.

Here \( \{u, x, y\} \) is a \( \mu \)-geodetic basis and \( \mu - gn(G) = 3 \).

**Definition 5.8** For a \( \mu \)-geodetic cover \( S \), a node in \( G \setminus S \) is called a \( \mu \)-check node.

**Remark 5.9** In crisp graphs [3] the unique geodetic basis of a tree consists of all its end nodes. But for a fuzzy tree \( \mu \)-geodetic basis need not be the set of fuzzy end nodes of \( G \).

**Example 5.10** Consider the fuzzy graph given in Fig.3.

Here fuzzy end nodes are \( v \) and \( w \). But \( \{v, w\} \) is not a \( \mu \)-geodetic cover, and \( \mu \)-geodetic basis is \( \{v, w, u\} \).

**Theorem 5.11** Let \( G : (V, \sigma, \mu) \) be a connected fuzzy graph on \( n \) nodes such that each pair of nodes in \( G \) is joined by a strong arc. Then \( \mu \)-geodetic number, \( \mu - gn(G) = n \).

**Proof**

Given \( G : (V, \sigma, \mu) \) be a connected fuzzy graph on \( n \) nodes such that each pair of nodes in \( G \) is joined by a strong arc. Then

\[
d_{\mu}(u, v) = \frac{1}{\text{CONN}_{\mu}(u, v)}
\]

for each arc \((u, v)\). [by Theorem 4.4]

Therefore no node lie on a \( \mu \)-geodesic between any two other nodes. Hence \( \mu \)-geodetic basis consists of all nodes of \( G \). Thus \( \mu - gn(G) = n \).

**Corollary 5.12** For a complete fuzzy graph \( G \), \( \mu - gn(G) = n \).

**Remark 5.13** Converse of Theorem 5.11 need not be true. If \( G : (V, \sigma, \mu) \) is a connected fuzzy graph on \( n \) nodes with \( \mu - gn(G) = n \), it does not imply that each pair of nodes in \( G \) is joined by a strong arc.

Consider the fuzzy graph given in Fig.2.

\( \mu - gn(G) = 4 \), But arc \((u, v)\) and arc \((u, w)\) are not strong arcs.
Theorem 5.14 For any connected fuzzy graph $G$, $\mu-\text{gn}(G)=2$ if and only if there exists $\mu-$peripheral nodes $u$ and $v$ such that every node of $G$ is on a $\mu-$peripheral path joining $u$ and $v$. Also let $P: u = u_0, u_1, u_2, ..., u_n = v$ be a $\mu-$peripheral path then

$$d_{\mu}(u,v) = d_{\mu}(u_0,u_1) + d_{\mu}(u_1,u_2) + d_{\mu}(u_2,u_3) + ... + d_{\mu}(u_{n-1},u_n).$$

Proof

Let $u$ and $v$ be such that each node of $G$ is on $\mu-$peripheral path $P$ joining $u$ and $v$. Since $G$ is nontrivial, $\mu-\text{gn}(G) \geq 2$. Since $P$ is a $\mu-$geodesic joining $u$ and $v$, each node of $G$ is on a $\mu-$geodesic between $u$ and $v$. So $S=\{u,v\}$ is a $\mu-$geodetic basis and $\mu-\text{gn}(G)=2$.

Conversely let $\mu-\text{gn}(G)=2$ and $S=\{u,v\}$ be a $\mu-$geodetic basis for $G$. To Prove that $d_{\mu}(G) = d_{\mu}(u,v)$.

Assume $d_{\mu}(u,v) < d_{\mu}(G)$.

Then $\exists$ $\mu-$peripheral nodes $s$ and $t$ such that $s$ and $t$ belong to distinct $\mu-$geodesics joining $u$ and $v$ and $d_{\mu}(s,t) = d_{\mu}(G)$.

Then, $d_{\mu}(u,v) = d_{\mu}(u,s) + d_{\mu}(s,v)$ ..... (1)

$$d_{\mu}(u,v) = d_{\mu}(u,t) + d_{\mu}(t,v)$$ ..... (2)

$$d_{\mu}(s,t) \leq d_{\mu}(s,u) + d_{\mu}(u,t)$$ ..... (3)

$$d_{\mu}(s,t) \leq d_{\mu}(s,v) + d_{\mu}(v,t)$$ ..... (4)

Since $d_{\mu}(u,v) < d_{\mu}(s,t)$

(3) $\implies$ $d_{\mu}(u,v) < d_{\mu}(s,u) + d_{\mu}(u,t)$ and by (1)

$d_{\mu}(s,v) < d_{\mu}(u,t)$ and from (4)

$d_{\mu}(s,t) < d_{\mu}(u,t) + d_{\mu}(v,t) = d_{\mu}(u,v)$ by (1), which is a contradiction. Thus $u$ and $v$ must be $\mu-$peripheral nodes.

Next Given $P: u = u_0, u_1, u_2, ..., u_n = v$ be a $\mu-$peripheral path. Since every node of $G$ is on $\mu-$peripheral path,

$$d_{\mu}(u_i-1, u_i) = \frac{1}{\mu(u_{i-1},u_i)}.$$

Therefore $d_{\mu}(u,v) = \min \{ \sum_{i=1}^{n} \frac{1}{\mu(u_{i-1},u_i)} \}.

= \sum_{i=1}^{n} d_{\mu}(u_{i-1}, u_i).

Therefore

$$d_{\mu}(u,v) = d_{\mu}(u_0,u_1) + d_{\mu}(u_1,u_2) + d_{\mu}(u_2,u_3) + ... + d_{\mu}(u_{n-1}, u_n).$$

Hence the proof.
Remark 5.15 In crisp graph $G$, if $v$ is a node that is farthest from $u$, then $v$ is not a cut node of $G$ [5]. But in fuzzy graphs, if $v$ is a node that is farthest from $u$ in $G$, then $v$ can be a fuzzy cut node of $G$. That is in fuzzy graphs fuzzy cut node can be $\mu$–eccentric node and $\mu$–peripheral node.

Example 5.16 Consider the fuzzy graph given in Fig.4.

\[ \text{Fig.4} \]

In Fig.4 $v$ is a fuzzy cut node and $v$ is an $\mu$–eccentric node of $w$ as well. Also $v$ is a $\mu$–peripheral node.

6 $\mu$–Convex Hull of a Fuzzy Graph

In this section $\mu$–convex hull and $\mu$–hull number of a fuzzy graph with respect to $\mu$–distance is defined.

Definition 6.1 Let $S \subseteq V(G)$ and repeatedly take its closures $S^1_\mu = (S_\mu), S^2_\mu = (S^1_\mu) = ((S_\mu))$ etc. Since we consider only fuzzy graphs with finite number of nodes, this process of taking closures must terminate with some smallest $n$ such that $S^n_\mu = S^{n-1}_\mu$. The resulting set is called $\mu$–convex hull of $S$ in $G$, and is denoted by $[S_\mu]$. 

Example 6.2 Consider the fuzzy graph given in Fig.1.

Here let $S = \{u, x, y\}$, which is not $\mu$–convex, and $\mu$–convex hull of $S$ in $G$, $[S_\mu] = \{u, v, w, x, y\}$.

Remark 6.3 It is clear from the definition that a subset $S \subseteq V(G)$ is $\mu$–convex if and only if $[S_\mu] = S$. Also $[S_\mu]$ is the smallest $\mu$–convex set containing $S$.

Definition 6.4 The minimum order of the set $S \subseteq V(G)$ such that $[S_\mu] = V(G)$ is called the $\mu$–hull number of $G$ denoted by $h_\mu(G)$ and such a set is called minimum $\mu$–hull set of $G$. 
Example 6.5 Consider the fuzzy graph given in Fig.1.

\[ S = \{u, x, y\} \]
\[ [S_\mu] = \{u, v, w, x, y\} = \text{V}(G) \]
\[ S = \{u, x, y\} \text{ is a minimum } \mu-\text{hull set.} \]
\[ h_\mu(G) = 3. \]

Remark 6.6 Let \( G : (V, \sigma, \mu) \) be a connected fuzzy graph with \( G^* \) complete and all arcs in \( G \) are strong. Then \( h_\mu(G) = n \).

Proposition 6.7 For a connected fuzzy graph \( G : (V, \sigma, \mu) \), \( 2 \leq h_\mu(G) \leq n \), where \( n \) is the number of nodes in \( G \).

7 Conclusion

In this paper, we introduced \( \mu \)-geodesic, \( \mu \)-eccentricity, \( \mu \)-radius, \( \mu \)-diameter, \( \mu \)-center, \( \mu \)-geodetic closure, \( \mu \)-geodetic iteration number, \( \mu \)-convex set, \( \mu \)-geodetic cover, \( \mu \)-geodetic basis, \( \mu \)-geodetic number, and \( \mu \)-convex hull of a fuzzy graph and studied some properties.

References


