On Mellin Transform Involving the Product of a General Class of Polynomials, Struve’s Function and H-Function of Two Variables

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Abstract

The object of this paper is to establish integrals involving the product of struve’s function, general class of polynomials and H-function of two variables. Some special cases have also derived.

Keywords: Mellin transform, struve’s function, general class of polynomials and H-function of two variables.

1 Introduction

Recently, The Mellin transform of struve’s function with H-function of two variables and Mellin transform of general class of polynomials with H-function of two variables [5] are evaluated. In the present paper we establish the Integral transform of H-function of two variables with general class of polynomials and struve’s function.

We shall utilized the following formulae in the present investigation. The
H-function of two variables given by Prasad and Gupta [7].

\[ H[x, y] = H^{M,N:m,n,g,h}_{P,Q:p,q,u,v} \left[ \gamma_{x}^{(a_{j} + \alpha_{j} A_{j}) \lambda_{H} P + (c_{j} + C_{j}) \lambda_{L} P + (e_{j} + E_{j}) \lambda_{U}} \right] \]

\[ = \frac{1}{(2\pi i)^{2}} \int_{L_{1}} \int_{L_{2}} \phi_{1}(s) \phi_{2}(t) \psi(s, t) x^{y} y^{x} ds \, dt, \quad i = \sqrt{-1} \]

where \( x, y \neq 0, \)

\[ \phi_{1}(s) = \prod_{j=1}^{m} \frac{\Gamma(d_{j} - D_{j} s)}{\Gamma(1 - c_{j} + C_{j} s)} \prod_{j=m+1}^{n} \frac{\Gamma(1 - d_{j} + D_{j} s)}{\Gamma(e_{j} - E_{j} s)} \]

\[ \phi_{2}(t) = \prod_{j=1}^{g} \frac{\Gamma(f_{j} - F_{j} t)}{\Gamma(1 - e_{j} + E_{j} t)} \prod_{j=g+1}^{h} \frac{\Gamma(1 - f_{j} + F_{j} t)}{\Gamma(e_{j} - E_{j} t)} \]

\[ \psi(s, t) = \frac{\prod_{j=1}^{M} \Gamma(b_{j} - \beta_{j} s - B_{j} t)}{\prod_{j=M+1}^{Q} \Gamma(1 - b_{j} + \beta_{j} s + B_{j} t)} \frac{\prod_{j=1}^{N} \Gamma(1 - a_{j} + \alpha_{j} s + A_{j} t)}{\prod_{j=N+1}^{P} \Gamma(1 - a_{j} + \alpha_{j} s - A_{j} t)} \]

where \( M, N, P, Q, m, n, p, q, g, h, u, v \) are all non negative integers such that \( 0 \leq N \leq P, Q \geq 0, 0 \leq m \leq q, 0 \leq n \leq q, 0 \leq g \leq v, 0 \leq h \leq u \) and \( a_{j}, \beta_{j}, A_{j}, B_{j}, C_{j}, D_{j}, E_{j}, F_{j} \) are all positive. The sequence of parameters \( (a_{P}, (b_{Q}), (c_{p}), (dq), (e_{u}) \) and \( (f_{v}) \) are so restricted that none of the poles of the integrand coincide.

The contour \( L_{1} \) lies in the complex s-plane and runs from \(-i\infty\) to \(+i\infty\) with loops, if necessary, to ensure that the poles of \( \Gamma(d_{j} - D_{j}s), (j = 1, 2, \ldots, m) \), lie to the right of the path; and those of \( \Gamma(1 - c_{j} + C_{j}s), (j = 1, 2, \ldots, n) \) and \( \Gamma(1 - a_{j} + \alpha_{j}s + A_{j}t), (j = 1, 2, \ldots, N) \) lie to the left of the path.

Also the contour \( L_{2} \) lies in the complex t-plane running from \(-i\infty\) to \(+i\infty\) with loops, if necessary, to ensure that the poles of \( \Gamma(f_{j} - F_{j}t), (j = 1, 2, \ldots, g) \), lie to the right of the path; and those of \( \Gamma(1 - e_{j} + E_{j}t), (j = 1, 2, \ldots, h) \) and \( \Gamma(1 - a_{j} + \alpha_{j}s + A_{j}t), (j = 1, 2, \ldots, N) \) lie to the left of the path. All poles of the integrand are simple poles.
Mellin transform of the H-function is defined as follows [12]

\[
\int_0^\infty x^{s-1} H_{P,Q}^{M,N}(x) \left[ \begin{align*}
\alpha_j & \gamma_j & h_j P
\delta_j & h_j Q
\end{align*} \right] \, dx = a^{-s} \theta(-s)
\]

where

\[
\theta(-s) = \frac{\prod_{j=1}^{N} \Gamma((1-c_j)-\gamma_j s)}{\prod_{j=1}^{M} \Gamma((1-d_j)+\delta_j s)} \prod_{j=N+1}^{P} \Gamma((1-c_j)+\gamma_j s) \prod_{j=M+1}^{Q} \Gamma((1-d_j)-\delta_j s)
\]

Provided the corresponding conditions stated in [12]
According Erdely [1, p.307]

\[
\int_0^\infty x^{s-1} \left[ \begin{align*}
\frac{1}{2\pi i} & \int_{c-i\infty}^{c+i\infty} g(s)x^{-s} dx
\end{align*} \right] \, dx = g(s)
\]

The Struve’s function is defined as [3],

\[
H_{\nu,v,u}[z] = \sum_{m=0}^{\infty} \frac{(-1)^m (z/2)^{v+2m+1}}{\Gamma(km+y)\Gamma(v+\lambda m+u)}
\]

\[\text{Re}(k) > 0, \text{Re}(\lambda) > 0, \text{Re}(v) > 0, \text{Re}(v+u) > 0\]

The class of polynomials [10]

\[
s_n^m[x] = \sum_{k=0}^{n} \frac{[n/m][-n]mk}{k!} A_{n,k}x^k \quad n = 0, 1, 2, \ldots
\]

where m is an arbitrary positive integer and the coefficients \(A_n, k\) \((n, k \geq 0)\) are arbitrary constants.

\section{Main Result}

\[
\int_0^\infty x^{s-1} H_{\nu,v,u}[a_j b_j c_j] s^m[b_j c_j d_j e_j f_j h_j \eta] \left[ \begin{align*}
\alpha_j & \beta_j & \gamma_j
\delta_j & \eta_j & \epsilon_j
\end{align*} \right] \, dx
\]
\[
\frac{1}{\delta} \sum_{j=0}^{\infty} \left[ \frac{n/m}{r} \sum_{r=0}^{F(r)} \theta \frac{\left( s+\phi(l,r) \right)}{\delta} H_{P+1+q2}^{N+2+q1+2} \left[ \begin{array}{c}
\begin{array}{c}
\sigma_j + \left( \frac{s+\phi(l,r)}{\delta} \right) A_j, a_j - (\sigma/\delta) A_j N_1^{c_j}, C_j M_{1,1}^{1,1}, m_{1,1}^{1,1} \\
i_j + \left( \frac{s+\phi(l,r)}{\delta} \right) B_j, b_j - (\sigma/\delta) B_j M_1^{C_j}, D_j M_{1,1}^{1,1}, q_{1,1}^{1,1} \end{array}
\end{array}
\right]
\end{array}
\right]
\]

where
\[
G(l) = \frac{(-1)^l d^{v+2l+1}}{\Gamma(kl+y)\Gamma(v+\lambda+u)}
\]

\[
F(r) = \frac{(-n)^r}{r!} A_{n,r} b^r
\]

\[
\phi(l,r) = g(v+2l+1)+hr
\]

Provided \( \sigma > 0, \delta > 0, \lambda > 0, h > 0, \)
\[
\alpha_j - (\sigma/\delta) A_j > 0 \text{ for } j = 1, 2, \ldots, P
\]
\[
\beta_j - (\sigma/\delta) B_j > 0 \text{ for } j = 1, 2, \ldots, Q
\]

\[|\arg \gamma| < (1/2) \pi \Delta_1, \quad |\arg \eta| < (1/2) \pi \Delta_2\]

where \( \Delta_1 = \sum_{j=1}^{N} \alpha_j - \sum_{j=N+1}^{N+1} \beta_j - \sum_{j=1}^{M} \beta_j + \sum_{j=M+1}^{M+1} \beta_j - \sum_{j=1}^{P} D_j + \sum_{j=P+1}^{P+1} D_j + \sum_{j=1}^{M} C_j - \sum_{j=M+1}^{M+1} C_j \)

\( \Delta_2 = \sum_{j=1}^{N} A_j - \sum_{j=N+1}^{P} A_j + \sum_{j=1}^{M} B_j - \sum_{j=M+1}^{M+1} B_j + \sum_{j=1}^{Q} F_j - \sum_{j=Q+1}^{Q+1} F_j + \sum_{j=1}^{M} E_j - \sum_{j=M+1}^{M+1} E_j \)

and
\[
\Re \left( s + g v + g + \frac{\delta(a_j-1)}{A_j} \right) < 0 \quad \text{for } j = 1, 2, \ldots, N
\]
\[
\Re \left( s + g v + g + \frac{\delta b_j}{B_j} \right) > 0 \quad \text{for } j = 1, 2, \ldots, M
\]
\[
\Re \left( s + g v + g + \frac{\delta f_j}{F_j} + \frac{\sigma d_j}{D_j} \right) > 0 \quad \text{for } j = 1, 2, \ldots, m_2; \text{ for } i = 1, 2, \ldots, m_1
\]
\[
\Re \left( s + g v + g + \frac{\delta e_j-1}{E_j} + \frac{\sigma(c_j-1)}{C_j} \right) > 0 \quad \text{for } j = 1, 2, \ldots, n_2; \text{ for } i = 1, 2, \ldots, n_1
\]
Proof.
We have
\[ H_{v, y, u}^{\lambda, k}[ax^k] e^{m_n[bx^h]} = \sum_{l=0}^{\infty} (-1)^m (ax^l/2)^{v+2l+1} \frac{[n/m](-n)!}{r!} A_{n, r} (bx^h) \]

Multiply both sides with
\[ x^{s-1} H_{P, Q}^{M, N: m_1, n_1, m_2, n_2} [a_j: \alpha_j, A_j; b_j: \beta_j, B_j; p; (e_j, E_j); (f_j, F_j)] \]
and integrate with respect to \( x \) from 0 to \( \infty \). By using (1.1), represent H-function in integral form and put \( \delta t = -w \). Interchange the order of integration then use result (1.3) and (1.2) to get the result. Change of order of integration is justifiable due to convergence of integrals.

3 Special Cases

Put \( h = 0, b = 1 \) in (2.1) we get Mellin transform of product of Struve’s function with H-function of two variables

(3.1) \[ \int_0^\infty x^{s-1} H_{v, y, u}^{\lambda, k}[ax^k] H_{P, Q}^{M, N: m_1, n_1, m_2, n_2} [a_j: \alpha_j, A_j; b_j: \beta_j, B_j; p; (e_j, E_j); (f_j, F_j)] dx \]

Put \( M = 0 \), we obtain

(3.2) \[ \int_0^\infty x^{s-1} H_{v, y, u}^{\lambda, k}[ax^k] e^{m_n[bx^h]} H_0^{0, N: m_1, n_1, m_2, n_2} [a_j: \alpha_j, A_j; b_j: \beta_j, B_j; p; (e_j, E_j); (f_j, F_j)] dx \]
\[
\frac{1}{\delta} \sum_{l=0}^{\infty} \sum_{r=0}^{n/m} \frac{[n/m]}{F(r)\eta} \left[ \frac{\sigma}{\delta} \frac{\eta}{\gamma} \left( a_j + \left( \frac{s+\phi(l,r)}{\delta} \right) A_j, \alpha_j - (\sigma/\delta) A_j \right)_{lN, (c_j, C_j)_{lN, (c_j, C_j)}} \right] \\
\left( 1-f_j - \left( \frac{s+\phi(l,r)}{\delta} \right) F_j, (\sigma/\delta) F_j \right)_{lQ_2, (c_j, C_j)_{lQ_2, (c_j, C_j)}} \\
\left( 1-e_j - \left( \frac{s+\phi(l,r)}{\delta} \right) E_j, (\sigma/\delta) E_j \right)_{lQ_2, (c_j, C_j)_{lQ_2, (c_j, C_j)}} 
\]

By taking M = N = P = Q = 0, we have

\[
(3.3) \quad \int_0^{s-1} H_{b_k, b_l}^{a_k, a_l} [b_k^h] S_n^m [b_l^h] H_{0,0}^{0,0} \cdot \eta \left[ \frac{\sigma}{\delta} \frac{\eta}{\gamma} \left( a_j + \left( \frac{s+\phi(l,r)}{\delta} \right) A_j, \alpha_j - (\sigma/\delta) A_j \right)_{lN, (c_j, C_j)_{lN, (c_j, C_j)}} \right] dx \\
= \int_0^{s-1} H_{b_k, b_l}^{a_k, a_l} [b_k^h] S_n^m [b_l^h] H_{0,0}^{0,0} \cdot \eta \left[ \frac{\sigma}{\delta} \frac{\eta}{\gamma} \left( a_j + \left( \frac{s+\phi(l,r)}{\delta} \right) A_j, \alpha_j - (\sigma/\delta) A_j \right)_{lN, (c_j, C_j)_{lN, (c_j, C_j)}} \right] dx 
\]

Put M = N = P = Q = 0 and (\alpha)j = (\beta)j = (A)j = (B)j = (C)j = (D)j = (E)j = (F)j = 1, we obtain

\[
(3.4) \quad \int_0^{s-1} H_{b_k, b_l}^{a_k, a_l} [b_k^h] S_n^m [b_l^h] H_{0,0}^{0,0} \cdot \eta \left[ \frac{\sigma}{\delta} \frac{\eta}{\gamma} \left( a_j + \left( \frac{s+\phi(l,r)}{\delta} \right) A_j, \alpha_j - (\sigma/\delta) A_j \right)_{lN, (c_j, C_j)_{lN, (c_j, C_j)}} \right] dx 
\]
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References