KREIN SPACE NUMERICAL RANGES: COMPRESSIONS AND DILATIONS

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Dedicated to Professor T. Ando, in recognition of his outstanding mathematical achievements

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Abstract. A criterion for the numerical range of a linear operator acting in a Krein space to be a two-component hyperbolical disc is given, using the concept of support function. A characterization of the Krein space numerical range as a union of hyperbolical discs is obtained by a reduction to the two-dimensional case. We revisit a famous result of Ando concerning the inclusion relation \( W(A) \subseteq W(B) \) of the numerical ranges of two operators \( A \) and \( B \) acting in (possibly different) Hilbert spaces, and the condition that \( A \) can be dilated to an operator of the form \( B \otimes I \). The extension of this result to operators acting in Krein spaces is investigated.

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