A CHARACTERISATION OF $C^*$-ALGEBRAS THROUGH POSITIVITY OF FUNCTIONALS

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Abstract. We show that a unital involutive Banach algebra, with identity of norm one and continuous involution, is a $C^*$-algebra, with the given involution and norm, if every continuous linear functional attaining its norm at the identity is positive.

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