STABILITY OF A FUNCTIONAL EQUATION OF WHITEHEAD ON SEMIGROUPS

VALERIY A. FAİZIEV1 AND PRASANNA K. SAHOO2∗

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ABSTRACT. Let $S$ be a semigroup and $X$ a Banach space. The functional equation $\varphi(xyz) + \varphi(x) + \varphi(y) + \varphi(z) = \varphi(xy) + \varphi(yz) + \varphi(xz)$ is said to be stable for the pair $(X,S)$ if and only if $f : S \to X$ satisfying $\|f(xyz) + f(x) + f(y) + f(z) - f(xy) - f(yz) - f(xz)\| \leq \delta$ for some positive real number $\delta$ and all $x, y, z \in S$, there is a solution $\varphi : S \to X$ such that $f - \varphi$ is bounded. In this paper, among others, we prove the following results: 1) this functional equation, in general, is not stable on an arbitrary semigroup; 2) this equation is stable on periodic semigroups; 3) this equation is stable on abelian semigroups; 4) any semigroup with left (or right) law of reduction can be embedded into a semigroup with left (or right) law of reduction where this equation is stable.


1 Tver State Agricultural Academy, Tver Sakharovo, Russia.
E-mail address: valeriy.faiz@mail.ru

2 Department of Mathematics, University of Louisville, Louisville, KY 40292 USA.
E-mail address: sahoo@louisville.edu

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∗ Corresponding author.
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