SOME RESULTS ON $\sigma$-DERIVATIONS

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Communicated by M. S. Moslehian

Abstract. Let $\mathcal{A}$ and $\mathcal{B}$ be two Banach algebras and let $\mathcal{M}$ be a Banach $\mathcal{B}$-bimodule. Suppose that $\sigma : \mathcal{A} \to \mathcal{B}$ is a linear mapping and $d : \mathcal{A} \to \mathcal{M}$ is a $\sigma$-derivation. We prove several results about automatic continuity of $\sigma$-derivations on Banach algebras. In addition, we define a notion for $m$-weakly continuous linear mapping and show that, under certain conditions, $d$ and $\sigma$ are $m$-weakly continuous. Moreover, we prove that if $\mathcal{A}$ is commutative and $\sigma : \mathcal{A} \to \mathcal{A}$ is a continuous homomorphism such that $\sigma^2 = \sigma$ then $\sigma d \sigma(\mathcal{A}) \subseteq \sigma(Q(\mathcal{A})) \subseteq \text{rad}(\mathcal{A})$.

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Date: Received: 17 July 2011; Accepted: 20 August 2011.

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2010 Mathematics Subject Classification. Primary 47B47; Secondary 17B40.
Key words and phrases. $\sigma$-derivation, derivation, $m$-weakly continuous linear mapping, quasi-nilpotent.