DIFFERENTIAL SUBORDINATIONS FOR CERTAIN ANALYTIC FUNCTIONS MISSING SOME COEFFICIENTS

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Communicated by M. S. Moslehian

Abstract. For a positive integer $n$, applying Schwarz’s lemma related to analytic functions $w(z) = c_n z^n + \cdots$ in the open unit disk $U$, some assertion in a certain lemma which is well-known as Jack’s lemma proven by Miller and Mocanu [J. Math. Anal. Appl. 65 (1978), 289–305] is given. Further, by using a certain method of the proof of subordination relation which was discussed by Suffridge [Duke Math. J. 37 (1970), 775–777] and MacGregor [J. London Math. Soc. (2) 9 (1975), 530–536], some differential subordination property concerning with the subordination

$$p(z) \prec q(z^n) \quad (z \in U)$$

for functions $p(z) = a + a_n z^n + \cdots$ and $q(z) = a + b_1 z + \cdots$ which are analytic in $U$ is deduced, and an extension of some subordination relation is given.

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Date: Received: 7 July 2010; Accepted: 27 August 2010.
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2010 Mathematics Subject Classification. Primary 30C80; Secondary 30C45.

Key words and phrases. Differential subordination, Schwarz’s lemma, Jack’s lemma, convex function, Briot–Bouquet differential equation.