ON AN EQUATION OF THE ELLIPSOID

By

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By Sir William R. Hamilton.

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The Secretary of Council read the following communication from Sir William Rowan Hamilton, on an equation of the ellipsoid.

"A remark of your’s, recently made, respecting the form in which I first gave to the Academy, in December, 1845, an equation of the ellipsoid by quaternions,—namely, that this form involved only one asymptote of the focal hyperbola,—has induced me to examine, simplify, and extend, since I last saw you, some manuscript results of mine on that subject; and the following new form of the equation, which seems to meet your requisitions, may, perhaps be shewn to the Academy tonight. This new form is the following:

\[ TV \frac{\eta \rho - \rho \theta}{U(\eta - \theta)} = \theta^2 - \eta^2. \] (1)

"The constant vectors \( \eta \) and \( \theta \) are in the directions of the two asymptotes required; their symbolic sum \( \eta + \theta \), is the vector of an umbilic; their difference, \( \eta - \theta \), has the direction of a cyclic normal; another umbilicar vector being in the direction of the sum of their reciprocals, \( \eta^{-1} + \theta^{-1} \), and another cyclic normal in the direction of the difference of those reciprocals, \( \eta^{-1} - \theta^{-1} \). The lengths of the semiaxes of the ellipsoid are expressed as follows:

\[ a = T\eta + T\theta; \quad b = T(\eta - \theta); \quad c = T\eta - T\theta. \] (2)

"The focal ellipse is given by the system of the two equations

\[ S \cdot \rho U\eta = S \cdot \rho U\theta; \] (3)

and

\[ TV \cdot \rho U\eta = 2S\sqrt{(\eta\theta)}; \] (4)

where \( TV \cdot \rho U\eta \) may be changed to \( TV \cdot \rho U\theta \); and which represent respectively a plane, and a cylinder of revolution. Finally, I shall just add what seems to me remarkable,—though I have met with several similar results in my unpublished researches,—that the focal hyperbola is adequately represented by the single equation following:

\[ V \cdot \eta \rho \cdot V \cdot \rho \theta = (V \cdot \eta \theta)^2. \] (5)