MA2341 - Advanced Mechanics 1
Michelmas Term - 2015-2016
Homework 9 - Due Dec. 15th, 2015

1. Show that the moment of inertia for a body consisting of several discrete masses m_i located at positions $\vec{x}_i$ can be written as

$$I_{m\ell} = -\sum_i m_i (R_i^2)_{m\ell},$$

where R_i is a 3×3 matrix with components

$$(R_i)_{mn} = \epsilon_{mnk}(\vec{x}_i)_k.$$

Furthermore, explicitly demonstrate the transformation properties of R_i under rotations and parity and classify it as an nth rank tensor or pseudotensor.

2. A homogeneous disk of radius R and mass M rolls without slipping (in the plane of the disk) on a horizontal surface. It is attracted to a force center located a distance d below the plane. Assume the force is proportional to the distance between the center of mass of the disk and the force center.

   (a) Calculate the moment of intertia of the disk relevant for this rolling.

   (b) Find the stable equilibrium position of the system and the frequency of small oscillations about that equilibrium.

3. Consider the rotational motion of a rigid body. Show that the components of the instanteous angular velocity expressed in the fixed coordinate system expressed in terms of the Euler angles are

$$\begin{aligned}
\omega_1' &= \dot{\theta}\cos\phi + \dot{\psi}\sin\theta\sin\phi \\
\omega_2' &= \dot{\theta}\sin\phi - \dot{\psi}\sin\theta\cos\phi \\
\omega_3' &= \dot{\psi}\cos\theta + \dot{\phi}
\end{aligned}$$

where $\vec{\omega} = \omega_1'\hat{x} + \omega_2'\hat{y} + \omega_3'\hat{z}$.