MA2341 - Advanced Mechanics 1
Michelmas Term - 2015-2016
Homework 7 - Due Dec. 1st, 2015

1. Consider a double pendulum with equal length massless rigid rods but different masses.

   (a) Find the normal frequencies and normal modes of small oscillation.

   (b) What happens if the lower mass is small compared to the upper one?

2. Consider a hoop of mass M attached to a pivot point. The hoop moves from side-to-side in its own plane like a pendulum. An additional bead of mass M is free to slide on the hoop. Note that the moment of intertia for the hoop is $I = 2MR^2$.

   (a) Find the general Lagrangian and the one for small oscillations.

   (b) Find any cyclic coordinates and conserved quantities.

   (c) Find the normal frequencies and normal modes.

   (d) Describe the motion of each of the normal modes.

3. Consider a mass m free to move on a frictionless table. The mass is attached to a spring of equilibrium length a which is free to rotate about a fixed pivot point.

   (a) Write the Lagrangian and determine any conserved quantities.

   (b) Sketch the effective potential and discuss its asymptotic behaviour.

   (c) Find the radius r_0 of circular orbits.

   (d) Find the frequency of small oscillations in r about r_0.