MA2341 - Advanced Mechanics 1
Michelmas Term - 2015-2016
Homework 6 - Due Nov. 24th, 2015

1. Consider the Kepler problem ($V = -k/r$) and the vector

$$\mathbf{A} = \mathbf{p} \times \mathbf{L} - mk\frac{\mathbf{r}}{r}.$$

   (a) Show that $\mathbf{A}$ is conserved.

   (b) For an elliptical orbit, give the magnitude and direction of $\mathbf{A}$.

   (c) How many conserved quantities exist for the Kepler problem? How many are independent?

2. Consider a particle of mass m moving in an elliptical orbit in the Kepler problem.

   (a) Express the semi-major axis of the ellipse in terms of the conserved quantities of the motion.

   (b) Do the same for the semi-minor axis.

   (c) Do the same for the orbital period and explicity demonstrate Kepler's Third Law, i.e. that $\tau^2 \propto a^3$.

3. Consider two particles interacting gravitationally and undergoing circular orbits around each other with period τ. Now assume they are instantaneously stopped and released. Prove that they collide after a time $\frac{\tau}{4\sqrt{2}}$. Hint: you may need the integral

$$\int \frac{y^2}{\sqrt{a-y^2}} dy = -\frac{y\sqrt{a-y^2}}{2} + \frac{a}{2}\sin^{-1}\left[\frac{y}{\sqrt{a}}\right].$$