MA2341 - Advanced Mechanics 1
Michelmas Term - 2015-2016
Homework 4 - Due Nov. 3rd, 2015

1. It is often difficult to get non-conservative forces into the Lagrangian formalism. However, we may always simply add a non-conservative component to the generalized force term in the Euler-Lagrange equations. Consider a system where the Cartesian components of the force have both conservative and dissipative parts:

$$F_i = -\frac{\partial V}{\partial x_i} + F_i^{diss},$$

where F_i^{diss} is not derivable from a potential. Suppose we wish to describe the system using $N \leq 3$ scleronomic generalized coordinates q_r, $r = 1, \dots N$, so that the Cartesian coordinates can be expressed as

$$x_i = x_i(q_r).$$

The Euler-Lagrange equations in the presence of this dissipative force now become:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_r} - \frac{\partial L}{\partial q_r} = Q_r^{diss}, \quad Q_r^{diss} = F_i^{diss}\frac{\partial x_i}{\partial q_r}.$$

   (a) Consider *Rayleigh's dissipation function* $K(\dot{x}_i)$ defined as

$$F_i^{diss} = -\frac{\partial K}{\partial \dot{x}_i}.$$

   Show that the Euler-Lagrange equations can be written as

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_r} - \frac{\partial L}{\partial q_r} + \frac{\partial K}{\partial \dot{q}_r} = 0.$$

   (b) Consider a simple pendulum moving in a viscous fluid such that the dissipative 'drag' force is

$$F_i^{diss} = -k\dot{x}_i.$$

   Find the dissipation function $K(\dot{\theta})$ and the equation of motion.

2. Consider a spherical pendulum with mass m and length ℓ. A spherical pendulum is like a simple pendulum, except the motion is no longer restricted to a plane.

(a) Find the lagrangian and equations of motion.

(b) What (if any) quantities are conserved?

(c) In what limit is the equation of motion for the ordinary plane pendulum recovered?

3. Consider the lagrangian for two degrees of freedom $\{q_1(t), q_2(t)\}$

$$L = \frac{m}{2}(a\dot{q}_1^2 + 2b\dot{q}_1\dot{q}_2 + c\dot{q}_2^2) - \frac{k}{2}(aq_1^2 + 2bq_1q_2 + cq_2^2)$$

where a,b, and c are arbitrary, but $b^2 - ac \neq 0$. Suppose q_1 and q_2 are scleronomous generalized coordinates, so that the two-dimenional cartesian coordinates can be expressed as

$$x_i = x_i(q_1, q_2),\ i = 1, 2.$$

(a) Determine such a coordinate transformation so that the lagrangian expressed in terms of $x_1(t)$ and $x_2(t)$ can be ‘decoupled’, i.e. written as

$$L[x_1, x_2, \dot{x}_1, \dot{x}_2] = L_1[x_1, \dot{x}_1] + L_2[x_2, \dot{x}_2].$$

(b) What physical system does this lagrangian describe? What is the purpose of the condition $b^2 - ac \neq 0$?