MA2341 - Advanced Mechanics 1
Michelmas Term - 2015-2016
Homework 3 - Due Oct. 27th, 2015

1. Consider a system with n degrees of freedom $\{y_i(x),\ i = 1, \ldots, n\}$ subject to the holonomic constraint $g(y_i, x) = 0$. Prove that the functions $y_i(x)$ which extremize the functional

$$J[y_i] = \int_{x_1}^{x_2} F[y_i, y_i', x]\, dx \tag{1}$$

subject to the constraint are given by

$$\frac{\partial \tilde{F}}{\partial y_i} - \frac{d}{dx}\left(\frac{\partial \tilde{F}}{\partial y_i'}\right) = 0,\ i = 1, \ldots, n \tag{2}$$

with $\tilde{F} = F + \lambda(x)g$ and $\lambda(x)$ a Lagrange multiplier.

2. Given the result of the previous question prove that for a system with n degrees of freedom $\{y_i(x),\ i = 1, \ldots, n\}$ subject to multiple holonomic constraints

$$g_j(y_i, x) = 0,\ j = 1, \ldots, s \tag{3}$$

(where $s < n$), the functions that extremize

$$J[y_i] = \int_{x_1}^{x_2} F[y_i, y_i', x]\, dx \tag{4}$$

subject to the constraints are described by

$$\frac{\partial \tilde{F}}{\partial y_i} - \frac{d}{dx}\left(\frac{\partial \tilde{F}}{\partial y_i'}\right) = 0,\ i = 1, \ldots, n \tag{5}$$

with $\tilde{F} = F + \sum_{j=1}^{s} \lambda_j(x) g_j$.

3. Determine initial conditions $(\theta_0, \dot{\theta}_0)$ for the simple plane pendulum such that the tension force vanishes at some point in the motion. Does this happen for all $(\theta_0, \dot{\theta}_0)$? If not, find conditions on $(\theta_0, \dot{\theta}_0)$ under which the tension force vanishes.

4. Consider a uniform hoop of mass m and radius r (moment of inertia: $I = mr^2$) which starts from rest and rolls without slipping off the top of a fixed hemisphere of radius R. At what angle does the hoop fall off of the cylinder?