MA2341 - Advanced Mechanics 1
Michelmas Term - 2015-2016
Homework 2 - Due Oct. 20th, 2015

1. Find the Lagrangian and equations of motion for a double pendulum consisting of equal masses m and identical massless rigid rods of length ℓ.

2. Obtain the Hamiltonian form of the Euler-Lagrange equations from their standard form. Consider a system with n degrees of freedom $\{q_i(t)\}$.

3. Consider 'snappy','jerky' dynamics (jerk is defined as the time derivative of acceleration, snap is the time derivative of jerk), a generalization of the Lagrange formalism in which the Lagrangian contains second and third time derivatives of the q_i as well as first time derivatives. By applying calculus of variations, show that the Euler-Lagrange equations become

$$-\frac{d^3}{dt^3}\left(\frac{\partial L}{\partial \dddot{q}_i}\right) + \frac{d^2}{dt^2}\left(\frac{\partial L}{\partial \ddot{q}_i}\right) - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) + \frac{\partial L}{\partial q_i} = 0, \qquad i = 1, 2, \ldots, n.$$

   Note that when affecting the variation, additionally the first and second time derivatives of the η_i vanish at the endpoints. How can this procedure be extended to obtain the Euler-Lagrange equations with arbitrary time derivatives in the Lagrangian?

   Apply this result to the Lagrangian

$$L = q(\gamma \dot{q} - \frac{m}{2}\ddot{q} - \frac{k}{2}q)$$

   to calculate the equation of motion. Do you recognize it?

4. (see Landau + Lifshitz, Chap. I, Prob. 2) Consider a simple pendulum of length ℓ and mass m_2. Suppose the point of support contains a mass m_1 and can move horizontally in the plane of pendulum's motion. Find the Lagrangian and equations of motion.