MA2341 - Advanced Mechanics 1
Michelmas Term - 2015-2016
Homework 1 - Due Oct. 13th, 2015

1. Prove

$$\boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{C}) = (\boldsymbol{A} \cdot \boldsymbol{C})\boldsymbol{B} - (\boldsymbol{A} \cdot \boldsymbol{B})\boldsymbol{C}$$

where $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ are arbitrary vectors, by using the following property of the Levi-Cevita symbols:

$$\epsilon_{ijk}\epsilon_{i\ell m} = \delta_{j\ell}\delta_{km} - \delta_{jm}\delta_{k\ell}.$$

2. Consider a scalar function of position and time $\phi(\boldsymbol{r}, t)$ and as well as a vector function of position and time $\boldsymbol{A}(\boldsymbol{r}, t)$. A particle moves in three dimensions under the influence of the (velocity- and time-dependent) potential

$$V = q\phi - q\dot{x}_i A_i$$

   (a) Find the Lagrangian and equations of motions for the particle.

   (b) Express the equations of motion in terms of

$$E_i = -\frac{\partial \phi}{\partial x_i} - \frac{\partial A_i}{\partial t}$$

   and

$$B_i = \epsilon_{ijk}\frac{\partial A_k}{\partial x_j}.$$

   Do they look familiar? Hint: you may need the identity from problem 1.

3. Consder the Lagrangian

$$L = \frac{m}{2}\dot{\boldsymbol{r}}^2 - V(\boldsymbol{r}).$$

(a) How does the change

$$L \to L + 2\boldsymbol{r} \cdot \dot{\boldsymbol{r}}$$

affect the equations of motion? Calculate explicitly the equations of motion with the changed and unchanged Lagrangian.

(b) More generally, for a system with degrees of freedom $\{q_i\}$ prove that the change

$$L \to L + \frac{df}{dt},$$

where $f(q, t)$ is an arbitrary scalar function, leaves the equations of motion invariant.