564 - Stochastic Methods Michelmas Term - 2013-2014 EXAM REVIEW TOPICS

• Mathematical Background:

- Single and multi-variable calculus
- Linear Algebra
- Basic Combinatorics

• Probability:

- sample space, probability
- conditional probability, Bayes' Theorem
- Discrete and continuous random variables
- Probability function, probability density function
- Mean, Variance, Covariance
- Distributions: Uniform, Gaussian, Exponential, Binomial, Poisson, Geometric

• Statistics:

- The Central Limit Theorem
- standard error, error propagation
- statistical significance
- Hypothesis testing: the χ^2 test
- Resampling techniques: jackknife and bootstrap

• Random number generation:

- Pseudo-random number generation
- The linear congruential generator: period
- RNG tests: The 'Birthday', 'Parking Lot', and 'Spectral' tests
- Generating discrete random variables:
 - * Discrete inverse transform method
 - * Finite number of choices with arbitrary probabilities
 - * Poisson, geometric, binomial
- Generating continuous random variables:
 - * Continuous inverse transform

- * uniform, exponential, cauchy distributions
- * Box-Muller and Marsaglia (polar) methods for gaussian.

• Monte Carlo integration:

- Integrals as expectation values of random variables
- variance reduction
- Importance sampling
- control variates
- stratified sampling: MISER and VEGAS

• Markov Chain Monte Carlo:

- Markov property, Homogeneous Markov Chain, Markov matrix
- stochastic and doubly-stochastic Markov matrices
- stationary states
- n-step transition probabilities
- first visit probability, total visit probability
- mean first passage time, mean recurrence time
- classification of states: positive recurrent, null recurrent, transient
- accessibility, communication, classes, irreducible
- period, aperiodic, ergodic
- fundamental limit theorem, thermalization, equilibrium
- autocovariance, autocorrelation, integrated autocorrelation time
- Modified central limit theorem, modified variance.
- reversibility, detailed balance
- Metropolis-Hastings algorithm: proposal matrix, acceptance matrix
- Gibbs sampler (Heatbath)
- The Ising model: Metropolis and Gibbs sampler
- Generalizations: Potts, O(N), scalar models
- Hybrid Monte Carlo: trajectory length, step size
- Reversible integrators, Leapfrog