5634 - Stochastic Methods
Michelmas Term - 2014-2015
Homework 2 - Due Nov. 12th, 2014

1. Consider a bank with 1000 customers. On average there are 50 withdrawal requests per month, while the number of withdrawals in a single month is Poisson distributed. On average, the amount of each withdrawal is 800€ and the amounts are exponentially distributed. Estimate the probability that the sum total of withdrawals in a given month exceeds 50,000€ by simulating approx. 50 million months.

2. Consider the integral

$$\int_0^3 \frac{e^{-s}}{1+\frac{s}{9}}\,\mathrm{d}s \approx 0.873109\,.$$

   (a) Estimate the value of this integral by generating uniform random numbers in the interval $[0,3]$. Tabulate your results for $10^2, 10^3, 10^4, 10^5$ random values. Make sure to estimate the error using the standard deviation.

   (b) Repeat the previous part using exponentially distributed random variables. Discuss which approach is more efficient and justify this conclusion.

3. Consider the integral

$$I = \int_{-10}^{10}\int_{-10}^{10} \frac{1}{2\pi}(1+x^2+y^2)^{-3/2}\,\mathrm{d}x\mathrm{d}y \approx 0.9103410982,$$

   The integrand is the probability density function for the bivariate Cauchy distribution, which has cumulative distribution function

$$F(x,y) = \frac{1}{4} + \frac{1}{2\pi}\left(\tan^{-1}x + \tan^{-1}y + \tan^{-1}\frac{xy}{\sqrt{1+x^2+y^2}}\right).$$

   (a) Obtain the exact answer given above analytically using the cumulative distribution function.

   (b) Use the GSL implementations of the MISER and VEGAS algorithms to solve the integral numerically. Perform some experiments to determine suitable parameters for each algorithm, and compare the results for a fixed number of function calls. Which is more efficient, MISER or VEGAS?